Problems - Shocks

(Solve only three out of the five problems!)

1. The upstream deHoffmann-Teller velocity is given by

$$\vec{V}_{HT} = -\frac{\vec{n} \times (\vec{B} \times \vec{V}_{in})}{\vec{n} \cdot \vec{B}}$$

Show that this is also the de Hoffmann-Teller velocity in the region downstream, i.e., that automatically the downstream flow is field-aligned when transforming into the upstream de Hoffmann-Teller frame.

2. Derive the following expression for the ratio of downstream to upstream tangential magnetic field component through a MHD discontinuity

$$\frac{(B_t)_2}{(B_t)_1} = r\frac{(v_n^2)_1 - (c_{int}^2)_1}{(v_n^2)_1 - r(c_{int}^2)_1}$$

where $r = (v_n)_1/(v_n)_2 = \rho_2/\rho_1$ is the compression ratio and $c_{int} = (B_n)_1/(\rho_1\mu_0)^{1/2}$ the upstream intermediate speed. Use for the derivation the tangential momentum jump condition and the condition that the tangential electric field is constant through the shock.

3. Cold solar wind ions impinge with the solar wind speed of 400 km/sec onto the bow shock at it's nose, i.e., parallel to the shock normal. The angle between the shock normal and the magnetic field is Θ_{Bn}. A few ions of the solar wind are reflected under conservation of the magnetic moment in the de Hoffmann-Teller frame. Assume that the solar wind velocity is much smaller than the de Hoffmann-Teller velocity. At which angle Θ_{Bn} is the velocity of the reflected particles 100 times the solar wind velocity (40000 km/sec)?

4. The interplanetary magnetic field is assumed to be aligned with the solar wind flow. The solar wind has a sonic mach number M_s and an Alfvén Mach number M_A. Use the fast magnetosonic wave speed to derive an expression for the asymptotic Mach cone angle $\alpha = \sin^{-1}(M)$, that is, determine M as a function of the sonic and Alfvénic Mach numbers.

5. Consider a fluid with negligible pressure which transports nonrelativistic energetic particles which are coupled to it by a constant diffusion coefficient κ. The fluid equations and the Parker equation for the particles are:

$$\frac{\partial \rho}{\partial t} = \nabla \cdot (\rho \vec{V}) = 0$$

$$\rho \frac{\partial V}{\partial t} + \rho(\vec{V} \cdot \nabla)\vec{V} = -\nabla P$$

$$\frac{\partial P}{\partial t} + \vec{V} \cdot \nabla P - \kappa \nabla^2 P + \gamma(\nabla \cdot \vec{V})P = 0$$

where P is the pressure of the energetic particles and $\gamma = 5/3$

(a) Consider a stationary planar system with variations in the x direction only and rewrite the equations for this system.

(b) Find three integrals of the system (mass flux, momentum flux, and energy flux conservation). To do this rewrite PdV/dx appearing in one term as $d/dx(PV) - VdP/dx$. In the terms involving dP/dx use the simplified version of the momentum equation to replace dP/dx by the term involving V and dV/dx,. The resulting equation can be integrated easily.

(c) Determine the three constants by setting $V = V_0 > 0$, $\rho = \rho_0$, and $P = 0$ as $x \to -\infty$.

(d) Derive the following equation for $V(x)$ alone by eliminating P in the energy flux integral:

$$\frac{2\kappa}{\gamma + 1} \frac{dV}{dx} = (V - V_0)\Big(V - \frac{\gamma - 1}{\gamma + 1} V_0\Big)$$

(e) Solve this equation for $V(x)$ and interpret the constant of integration. Derive expressions for $\rho(x)$ and $P(x)$ and plot them schematically. What does the solution represent?