An empirical formula relating the geomagnetic storm’s intensity to the interplanetary parameters: $-\nabla B_z$ and $\Delta t$

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[1] We statistically study 105 geomagnetic storms with a $\text{Dst}$ peak value $\leq -50$ nT during 1998–2001 to examine the influence of the interplanetary parameters $-\nabla B_z$ and its duration $\Delta t$ on the intensity of geomagnetic storms. About 33% of the events are associated with intense storms with $\text{Dst}_{\text{min}} \leq -100$ nT. It is found that $-\nabla B_z$ is much more important than $\Delta t$ for the formation of geomagnetic storms. A stronger $-\nabla B_z$ can produce a more intense storm, whereas a longer $\Delta t$ can not. A simple empirical formula relating the $\text{Dst}$ peak value to $-\nabla B_z$ and $\Delta t$ is obtained, which shows a good correlation ($CC = 0.9528$) between the estimate value and the observations. This formula suggests that a compressed $B_z$ field tends to have a more prominent geoeffectiveness. Moreover, we also identify 33 large $\text{Dst}_{\text{min}}$ storms ($\text{Dst}_{\text{min}} \leq -200$ nT) and 8/9 of the great storms ($\text{Dst}_{\text{min}} \leq -200$ nT) were due to interplanetary compressed structures.


1. Introduction

[2] A geomagnetic storm is due to the enhancement of ring current at the Earth’s equator. $\text{Dst}$ is one of the important indices in evaluating the level of geomagnetic disturbances. The interplanetary causes of geomagnetic storms have been extensively studied [e.g., Tsurutani et al., 1997 and therein]. The Earth-directed solar wind speed ($V$) and southward component ($B_z$) of interplanetary magnetic fields are of most importance in creating geomagnetic storms [Snyder et al., 1963; Fairfield and Cahill, 1966]. Certainly, a sufficiently long duration of $B_z$ is also necessary.

[3] Based on statistical studies, the interplanetary criteria in creating geomagnetic storms has been concluded. For intense storms with $\text{Dst}_{\text{min}} \leq -100$ nT, the value of $B_z$ should be greater than 10 nT with its duration $\Delta t \geq 3$ hours [Gonzalez and Tsurutani, 1987]. For moderate storms with $\text{Dst}_{\text{min}} \leq -50$ nT, the threshold values of $B_z \geq 5$ nT and $\Delta t \geq 2$ hours were suggested [Russell et al., 1974]. Moreover, Cane et al. [2000] found a clear correlation between $B_z$ within ejecta or sheath regions and $\text{Dst}_{\text{min}}$ with a correlation coefficient of 0.74. Recently, Wu and Lepping [2002] further confirmed the conclusion that there is a good correlation between $\text{Dst}_{\text{min}}$ and the solar wind parameters, $(V B_z)_{\text{min}}$ and $B_{z,0}$, by investigating the geomagnetic activities associated with magnetic clouds on the basis of the WIND observations.

[4] To predict the $\text{Dst}$ value, Burton et al. [1975] proposed a simple equation for the evolution of $\text{Dst}^*$ (pressure corrected measured $\text{Dst}$) in terms of solar wind conditions: $\frac{d\text{Dst}^*}{dt} = Q(t) - \text{Dst}^*$, where $Q$ (commonly $VB_z$ is adopted) is the coupling function and $\tau$ is the decay time. Based on this relationship, several of models have been developed to estimate the evolution of $\text{Dst}$ [e.g., Fenrich and Luhmann, 1998; O’Brien and McPherron, 2000a, 2000b; Lundstedt et al., 2002]. Obviously, the last term in the above equation suggests that the loss effect becomes significant and inhibits the growth of the storm if the storm takes a long time to develop [e.g., Daglis et al., 1999; O’Brien and McPherron, 2000a]. However, a general relationship between the intensity of the $B_z$ field (or the $-\nabla B_z$ electric field) and its duration $\Delta t$ as a function of storm intensity $\text{Dst}_{\text{min}}$ has not been found yet.

[5] The main aim of this letter is to find out a direct relationship between $-\nabla B_z$, $\Delta t$ and $\text{Dst}_{\text{min}}$, and to reveal the importance of the interplanetary parameter $-\nabla B_z$ and its duration $\Delta t$ to the geomagnetic storms. Previous work [Burton et al., 1975] suggested that the duration with $-\nabla B_z < 0.5$ mV/m is of little geoeffectiveness. Therefore, here, $\Delta t$ is the interval where $-\nabla B_z \geq 0.5$ mV/m from the occurrence of the $B_z$ field, which causes the storm, to the corresponding $-\text{Dst}$ peak, and $-\nabla B_z$ is the average value of $-\nabla B_z$ during this interval. We statistically analyze 105 geomagnetic storms with $\text{Dst}_{\text{min}} \leq -50$ nT during 1998–2001. In the next section, we will introduce the selection of the sample and the method. The results are presented in Section 3. Finally, we discuss the results and give a summary.

2. Data and Method

[6] In this letter, we only select the moderate to intense storms ($\text{Dst}_{\text{min}} \leq -50$ nT), in which the contribution of interplanetary causes is prominent. Sometimes geomagnetic storms have double or triple $\text{Dst}$ peaks [e.g., Kamide et al., 1998; Jordanova et al., 2003]. In such events, the latter $\text{Dst}$ peak is produced based on the former $\text{Dst}$ storm. These multiple-step storms might influence the statistical results, so we only consider the first $\text{Dst}$ peak, and exclude other $\text{Dst}$ peaks for the multiple-step storms. The interplanetary observations from the ACE spacecraft are used due to its relatively fixed orbit at L1 libration point. During the ACE data gap, the WIND observations are used instead.
According to above criteria, a total of 105 events are selected from 1998 to 2001, among which there are 35 intense storms with $D_{st\min} \leq -100$ nT. Although the ACE spacecraft is $\sim 10^6$ km away from the magnetopause, the delay time, which is approximately less than 1 hour, is not considered, because of the 1-hour resolution of $D_{st}$. The following two examples are represented to explain the method of dealing with the data in detail.

### 2.1. October 29, 2000 Event

[7] Figure 1 shows the ACE observations of this event. The sudden commencement (SC) of the geomagnetic storm was at about 2000 UT on Oct. 28. Obviously, the $B_z$ interval denoted by the vertical dashed lines should be responsible for the formation of this storm. The amplitude of the magnetic field increased suddenly from 8 nT to 20 nT, i.e., $B_{z_f}$ decreased to nearly $-20$ nT and formed a large $B_z$ interval. $VB_z$ decreased from a positive value to a negative value accordingly. Within this interval, only the durations, in which the value of $-VB_z$ was larger than 0.5 mV/m, have contribution to the storm as denoted by the filled region. Although there was still a long duration with $-VB_z \geq 0.5$ mV/m after the $D_{st}$ peak, it is not included, because that $-VB_z$ duration had nothing to do with the peak. In this event, the value of $-VB_z$ is 6.40 mV/m, and the duration $\Delta t$ is 7.15 hours. $\Delta t$ is shorter than the time from the beginning of the $B_z$ interval to the $D_{st}$ peak.

### 2.2. November 7–8, 1998 Event

[8] Figure 2 shows the ACE observations of this event. This geomagnetic storm has a double-peak structure (as marked by ‘1’ and ‘2’). A long $B_z$ interval, which caused the first $D_{st}$ peak ($\sim 81$ nT) at 1700 UT, began at 1100 UT approximately on Nov. 7. In the same way, we can obtain that $-VB_z = 3.57$ mV/m and $\Delta t = 5.70$ hours. The second peak arrived at 0700 UT on Nov. 8. However, it was influenced by the former peak obviously because the background value of the second peak was about $-50$ nT, which largely deviated from the value at the quiet time. Thus, to avoid such influence, the second $D_{st}$ peak is excluded from our sample though its peak value is much larger than the first.

### 3. Results

[9] The distributions of $\Delta t$, $-VB_z$ and $-\overline{VB}_z$ for the geomagnetic storms are shown in Figure 3, respectively. The upper row presents the situation for the storms with $D_{st\min} \leq -50$ nT. The duration $\Delta t$ is not shorter than 1 hour, and 103 of 105 (98%) events are associated with $\Delta t \geq 2$ hours. A majority (63%) of the events are concentrated in the region of 3 hours $\leq \Delta t < 11$ hours. The peak of the distribution appears at $\Delta t \sim 6$ hours. The average value of $B_{z_f}$ is not smaller than 3 nT, and 89 of 105 (85%) events are associated with $\overline{B}_z \geq 5$ nT. Almost all (95%) of the events are concentrated in the region of 3 nT $\leq \overline{B}_z < 16$ nT. The $-\overline{VB}_z$ distribution peak is located at about 6 nT. The value of $-\overline{VB}_z$ is larger than 1 mV/m, and 94 of all (90%) events are associated with $-\overline{VB}_z < 7$ mV/m. The largest probability appears at $-\overline{VB}_z \sim 3$ mV/m.

[10] In our sample, 35 of 105 ($\sim 33$%) events are associated with intense storms ($D_{st\min} \leq -100$ nT). The situation of such large events is similar with the former except for a small right-shift of the distributions (shown in the lower row in Figure 3). $\Delta t$ is longer than 2 hours, and the peak is at about 7 hours. The value of $-\overline{VB}_z$ is larger than 6 nT, and the peak appears at $\sim 10$ nT. The value of $-\overline{VB}_z$ is larger than 3 mV/m, and the peak is located at 5 mV/m approximately. Obviously, all of the parameters shift toward the larger values, i.e., the more intense storms should be due to the larger $-\overline{VB}_z$ intervals.

[11] Figure 4 shows the relationship between $-\overline{VB}_z$, $\Delta t$ and $D_{st\min}$ for all the events. From the upper panel, it is found that the moderate storms are scattered over a large range from 1 hour to 28 hours, whereas the great storms with $D_{st\min} \leq -200$ nT are concentrated in a more narrow range from 2 hours to 14 hours approximately. This result implies that the longer duration is not necessary to produce the larger geomagnetic storm, because the effect of energy...
loss becomes more prominent [e.g., Daglis et al., 1999]. On
the other hand, a good linear anti-correlation with
\( CC = 0.9147 \) between \( \Delta \tau \) and \( Dst_{min} \) is presented in the
second panel. The larger \( \Delta \tau \) is, the more intense is the
geomagnetic storm.

[12] Generally, \( \Phi = -VB_z \Delta \tau \) is considered the magnetic
flux transferred from interplanetary medium into the inner
magnetosphere. However, the linear correlation \( (CC = 0.7226) \) between
\( VB_z \) and \( Dst_{min} \) is weaker. Obviously, the non-linear relation between
\( VB_z \) and \( Dst_{min} \) weakens the correlation. The weights of \( VB_z \) and \( \Delta \tau \)
should be different. We therefore use the variable \( VB_z \alpha (\Delta \tau)^{\beta} \), where \( \alpha \) and \( \beta \) are tunable constants, to linearly fit the
\( Dst \) data. It is found that the following empirical formula

\[
Dst_{min} = -19.01 - 8.43 (\Delta t)^{0.30} \text{nT}
\]

has a minimum anti-correlation coefficient of \(-0.9528\), which indicates a better goodness-of-fit, as shown in the
fourth panel of Figure 4.

4. Discussion and Summary

[13] The distributions represented in the last section give the
threshold values of \( B_z \geq 3 \) nT and \( \Delta \tau \geq 1 \) hour for moderate geomagnetic storms with \( Dst_{min} \leq -50 \) nT, and the threshold values of \( B_z \geq 6 \) nT and \( \Delta \tau \geq 2 \) hours for intense storms with \( Dst_{min} \leq -100 \) nT. In our statistical
study, the investigated interval is from the occurrence of the
\( B_z \) field, which causes the geomagnetic storm, to the \( Dst \)
peak, and an average value of \( B_z \) is adopted. Hence, these
threshold values are all different from, actually smaller than,
the results obtained by Gonzalez and Tsurutani [1987] and
Russell et al. [1974], in whose works the entire \( B_z \) interval
and the maximum value of \( B_z \) were used. In addition, the
distributions for all the storms seem to follow the \( \Gamma \)
distribution \( (CC > 0.90) \) as shown in Figure 3. As for the
intense storms, we do not try to fit them by \( \Gamma \) distribution
function, because the number of the events is too small.

[14] We also obtain a good anti-correlation between
\( Dst_{min} \) and \( -VB_z \) on the basis of a larger sample. The result
is consistent with that obtained by Wu and Lepping [2002]
except that the average value of \( -VB_z \) is used here. Wu and
Lepping [2002] studied the events for solar minimum
(1995–1998). According to our result, such good correla-
tion is also suitable for the ascending phase and the peak of

[15] Empirical formula 1 shows that the weight of \( \Delta \tau \) is
less than that of \( -VB_z \). For a fixed \( -VB_z \), the value of
\( (Dst_{min} + 19.01) \propto (\Delta \tau)^{0.30} \). Thus, a long duration is not
very helpful to further enhance a storm’s intensity. On the
contrary, a large \( -VB_z \) has a prominent contribution to
creating a strong geomagnetic storm. The point ‘A’ labelled
in Figure 4 denotes such a event associated with large
\( -VB_z = 20.81 \) mV/m but short \( \Delta \tau = 4.90 \) hours on March
31, 2001, which produced the largest geomagnetic storm with $\text{Dst}_{\text{min}} = -387$ nT. This special event has been studied by Wang et al. [2003a] recently. During this event, a multiple magnetic cloud [Wang et al., 2002], in which a extraordinary large $B_s$ field was formed due to the sub-clouds’ compression, passed through the Earth.

[16] Assuming the magnetic flux $\Phi = -VB_s\Delta t = \text{constant}$, we can rewrite the formula 1 as $\text{Dst}_{\text{min}} = -19.01 - 8.43\Phi^{0.30} (-\text{VB}_s)^{0.79} = -19.01 - 8.43\Phi^{1.09} (\Delta t)^{-0.79}$. As is well known, a compressed $B_s$ interval is associated with a larger $-\text{VB}_s$ and a shorter $\Delta t$ than its original state. If $\Delta t$ is shortened to a half, and $-\text{VB}_s$ enhances 1 time accordingly, the value of $(\text{Dst}_{\text{min}} + 19.01)$ is therefore 1.73 times its original value. Thus, the compressed $B_s$ field should have a more prominent geoeffectiveness than the original field. This is suggests that the multiple magnetic cloud and the shock compression of preexisting southward magnetic fields tend to create the larger geomagnetic storms [Tsurutani et al., 1992; Wang et al., 2003b]. In addition, we investigate the large $-\text{VB}_s$ intervals during 1998–2001 using the ACE observations. There were 33 large $-\text{VB}_s$ intervals with $-\text{VB}_s \geq 5$ mV/m and $\Delta t \geq 3$ hours identified. They all caused intense geomagnetic storms ($\text{Dst}_{\text{min}} \leq -100$ nT), among which there were 9 great storms ($\text{Dst}_{\text{min}} \leq -200$ nT). Especially, almost all (8/9) of the great storms were due to compressed interplanetary structures.

[17] The same linear fit is done for the 35 large storms (as shown in the bottom panel of Figure 4). Another formula: $\text{Dst}_{\text{min}} = -66.31 - 3.21\Phi (-\text{VB}_s)^{1.35} (\Delta t)^{0.33}$ has the best correlation coefficient. It may be estimated that the error between the two formulae is approximately less than 3% when $-150$ nT $<$ $\text{Dst}_{\text{min}}$ $<$ $-500$ nT. The values of $\alpha$ and $\beta$ suggest that the weight of $-\text{VB}_s$ relative to $\Delta t$ increases, and the loss effect becomes more prominent during a large storm.

[18] To directly relate the interplanetary observations with the ground-based measurements, we use Dst rather than Dst*. Actually, the solar wind pressure will influence Dst. So we examine the relationship by using Dst* as well. It is found that the tunable constants $\alpha$ and $\beta$ are 1.06 and 0.27 respectively, approaching those derived for Dst, and a high correlation ($CC = -0.93$) is also obtained.

[19] Actually, the idea that $VB_s$ is more important than $\Delta t$ has been impliedly presented in Burton et al. [1975] work. By integrating Burton et al. [1975] equation (also seen in Introduction), one can get a relationship: $\text{Dst}_{\text{min}} * = e^{-\Delta t} \int_0^\Delta t \text{VB}_s \Delta t e^{-\Delta t} dz = -\text{VB}_s (1 - e^{-\Delta t})$. Using the form: $\text{Dst}_{\text{min}} = k_0 + k_1\text{VB}_s (1 - e^{-\Delta t})$ to fit the observations, we also obtain a high correlation coefficient of 0.9490 with $k_0 = -13.62$, $k_1 = 22.25$ and $\tau = 3.10$. This formula shows that the $\text{Dst}_{\text{min}}$ is described as a family of hyperbolas, and suggests that a $B_s$ interval with a long duration cannot further produce a much larger storm. Compared to it, formula 1 shows a simple and direct relationship between geomagnetic storm’s intensity and interplanetary parameters. For the most storms, the two formulae are comparable. However, for some large storms associated with very short or very long $B_s$ intervals, e.g., the events ‘B’ ($-\text{VB}_s = 13.11$, $\Delta t = 2.55$ and $\text{Dst}_{\text{min}} = -201$) and ‘C’ ($-\text{VB}_s = 5.40$, $\Delta t = 23.30$ and $\text{Dst}_{\text{min}} = -149$) marked in Figure 4, the error of Burton et al. [1975] formula seems to be much larger.

[20] In summary, we have statistically studied the relationship between the Dst peak value, $-\text{VB}_s$ and $\Delta t$, and obtained a simple empirical formula, which is very consistent with the observations. Our analyses do not concern the interplanetary origin of the $B_s$ interval. These results are significant in understanding of geomagnetic storms. It is suggested again that the weight of $-\text{VB}_s$ is much larger than that of $\Delta t$. This conclusion can illuminate that the compressed southward magnetic field has a larger geomagnetic storms. However, whether and how one can use the formula to predict the evolution of Dst should be studied further.

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