Existing an Upper Limit in the Magnetic Energy of a Stable Magnetic Flux Rope?

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Magnetic flux rope (MFR) is a fundamental structure in the universe filled with plasmas, and is the core of various eruptive phenomena, such as coronal mass ejections (CMEs), solar jets and astrophysical jets. Twist is a key parameter characterizing a MFR, which is described as

$$T = \frac{B_\phi}{rB_z} \quad (1)$$

in a cylindrical coordinates (r, ϕ, z). The total twist angle is given by

$$\Phi = \int_0^l T dz = Tl \quad (2)$$

for a uniformly twisted MFR, in which l is the length of the MFR.

The previous theoretical work (Dungey & Loughhead 1954) and the statistical study on 115 interplanetary magnetic clouds at 1 au (Wang et al. 2016) have suggested that a MFR probably becomes unstable when Φ exceeds a critical value

$$\Phi_c = 2\frac{l}{R} \quad (3)$$

where R is the radius of the MFR. This reaches an inference that (1) a thinner and/or longer MFR can be higher-twisted than a thicker and/or shorter MFR, and (2) the inner core of a MFR can be more twisted than the outer shell.

Assuming the above picture is real and plasma density in a MFR will keep unchanged when twisting or untwisting it, we may conjecture that there exists an upper limit, $W_{B,c}$, in the magnetic energy, W_B, of a stable MFR, and the upper limit is five times of the magnetic energy, $W_{B,0}$, of the MFR having been untwisted, i.e.,

$$\frac{W_{B,c}}{W_{B,0}} = 5 \quad (4)$$

or

$$\frac{W_B}{W_{B,0}} \leqslant 5 \quad (5)$$

Proof: Considering a MFR, in which the magnetic field lines have been uniformly twisted to the critical value, we may derive that the length of the magnetic field lines on any torus is

$$\begin{aligned} l_{\text{line},c} &= \int_0^l \sqrt{dz^2 + (rd\phi)^2} = l\sqrt{1 + r^2 T_c^2} \\ &= l\sqrt{1 + \frac{r^2}{l^2}\Phi_c^2} = \sqrt{5}\, l \end{aligned} \quad (6)$$

Under the large magnetic Raynolds number approximation, which is the case for a MFR in the solar corona and interplanetary space, it can be derived that

$$\frac{\boldsymbol{B}}{\rho} \propto \boldsymbol{l}_{\text{line}} \quad (7)$$

where ρ is the plasma density. The above relation implies that the magnetic field strength linearly increases as the field line is stretched under the assumption of constant density. We then can compare the magnetic field of a MFR at the critical state with that of the MFR completely untwisted, of which the magnetic field lines are all straight with a length of *l*, i.e.,

$$\frac{B_c}{l_{\text{line},c}} = \frac{B_0}{l} \Rightarrow B_c = \sqrt{5} B_0 \quad (8)$$

The magnetic energy is

$$W_{B,c} = \int \frac{B_c^2}{2\mu} dV = \int \frac{5B_0^2}{2\mu} dV = 5W_{B,0} \quad (9)$$

End of Proof.

This conjecture might be tested by studying the solar MFRs, i.e., CMEs, because we have the vector magnetic field observations at the photosphere and are able to extrapolate the coronal magnetic field from the photospheric magnetograms to search possible MFRs in the solar corona.

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