Effect of Electron Drift Velocity on Whistler Instability in Collisionless Magnetic Reconnection *

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The whistler instability is studied under the condition that the electron and ion velocities can be described in a bi-Maxwellian distribution with a field-aligned electron outflow drift velocity. It is found that the electron outflow drift velocity might obviously make the threshold condition of whistler instability decrease when this velocity is parallel to the magnetic field, whereas the electron outflow drift velocity might increase the threshold condition when this velocity is anti-parallel to the magnetic field in collisionless magnetic reconnection.

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It is well known that magnetic reconnection is a main process in providing a mechanism for the fast release of stored magnetic energy in solar flare, magnetosphere and magnetopause. In the resistive magnetic reconnection model,[1,2] the anomalous resistivity is provided by the wave activity and makes the electrons and ions no separation, and the magnetic field are controlled by the motion of ions. However, this model was unable to explain satisfactorily the short timescale for a rapid magnetic reconnection, and there was no evidence of such anomalous resistivity. On the other hand, a collisionless magnetic reconnection model was suggested:[3–6] the motion of electrons and of ions decouples, and the magnetic field mainly moves with the electrons' motion. The whistler waves are basically supported by the motion of electrons and hence have a much faster velocity for small-scale perturbation. The reconnection rate becomes larger. The whistler waves have been observed in situ spacecraft GEOTAIL during the magnetic reconnection along the dayside magnetopause.[7] Meanwhile, these whistler waves have also been observed during the magnetic reconnection in a laboratory plasma.[8] It is of interest, in this Letter, to study how the whistler waves are excited in the magnetic reconnection.

Figure 1 shows a sketch of a magnetic reconnection process. The electrons are accelerated by Fermi acceleration during the inflow of the magnetic line close together in the magnetic reconnection; this will enhance the perpendicular component of electron velocity. However, such a Fermi acceleration process for ions is unavailable because ions have a large cyclotron radius. After the magnetic reconnection, the electrons and ions in the outflow move from o point to both the sides, as shown in Fig. 1. The magnetic field from point o to points a, b, c and d looks like a magnetic mirror, the parallel component of velocity transforms to the perpendicular component of the velocity, when the electrons and ions move from point o to those points. Therefore, the electron and ion velocity distribution should be anisotropic with a field-aligned outflow drift velocity during the magnetic reconnection process. Such an anisotropic velocity distribution has been identified from the simulation of collisionless magnetic reconnection.[5,6,15] It is well known that whistler waves may be excited by a bi-Maxwellian electron velocity distribution plasma, if the perpendicular temperature component is larger than the parallel temperature component in the electron velocity distribution. This is the whistler instability that has been discussed by Refs. [9–12] However, the authors did not take the influence of the field-aligned outflow drift velocity into account in their papers. We find that the field-aligned outflow drift velocity is important for the threshold condition of whistler instability; it depends on whether the field-aligned outflow drift velocity is in parallel or anti-parallel to the magnetic field in the collisionless magnetic reconnection.

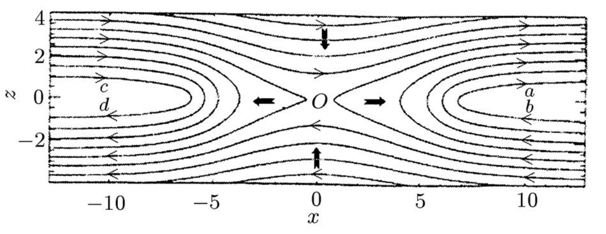

Fig. 1. Sketch of magnetic reconnection.

In the frame of wave vector $\boldsymbol{k}$ taken in the plane of (x,z), and $k_y = 0$, the dielectric tensor ϵ_{ij} of a hot magnetized homogeneous plasma can be expressed

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as[13]

$$\epsilon_{ij} = \delta_{ij} + \sum_S \frac{4\pi e_s^2}{m_s\omega^2}\Big[\sum_{\ell=-\infty}^{\infty}\int d^3v\Big(\frac{\omega - kv_\parallel}{v_\perp}\frac{\partial f_{s0}}{\partial v_\perp} + k\frac{\partial f_{s0}}{\partial v_\parallel}\Big)\frac{\Pi_{ij}^{(s\ell)}}{\omega - kv_\parallel - \ell\Omega_s} - \Big(n_s + \int\frac{v_\parallel^2}{v_\perp}\frac{\partial f_{s0}}{\partial v_\perp}d^3v\Big)b_z b_z\Big], \quad (1)$$

where

$$\Pi_{ij}^{(s\ell)} = \begin{vmatrix} \frac{\ell^2\Omega_s^2}{k_x^2}J_\ell^2 & i\frac{\ell\Omega_s v_\perp}{k_x}J_\ell J_\ell' & \frac{v_\parallel \ell\Omega_s}{k_x}J_\ell^2 \\ -i\frac{\ell\Omega_s v_\perp}{k_x}J_\ell J_\ell' & v_\perp J_\ell'^2 & -iv_\perp v_\parallel J_\ell J_\ell' \\ \frac{v_\parallel\ell\Omega_s}{k_x}J_\ell^2 & iv_\perp v_\parallel J_\ell J_\ell' & v_\parallel^2 J_\ell^2 \end{vmatrix}, \quad (2)$$

where $v_\parallel$ and $v_\perp$ are velocity components in the parallel and perpendicular direction of magnetic field, respectively. $\Omega_s = \frac{e_s B_0}{m_s c}$ and ω_{ps} are cyclotron frequency and plasma frequency of particle s, respectively; n_s is the density of particle s. $J_\ell = J_\ell(\lambda_s)$, $J_\ell'(\lambda_s) = \frac{dJ_\ell}{d\lambda_s}$ are the Bessel functions and their derivatives; $\lambda_s = \frac{k_x v_\perp}{\Omega_s}$, and $b_z = \frac{|\boldsymbol{B}_0|}{B_0}$ is a unit vector along the external magnetic field. For simplicity, we only discuss the excited wave propagation along the magnetic field, $k_\parallel = k$, $k_x = k\sin\theta = 0$, or $\theta = 0$ and $\lambda_s = 0$; while θ is an angle between the magnetic field and the wave propagation direction. In this case, $J_\ell(0) = \delta_{\ell 0}$, the dispersion equation for transverse wave with $k_\perp = 0$ can be written as

$$\begin{aligned}\Lambda(k,\omega) &= \epsilon_{xx} \mp i\epsilon_{xy} - \frac{k^2c^2}{\omega^2} \\ &= 1 - \frac{k^2c^2}{\omega^2} + \sum_s \frac{\omega_{ps}^2}{\omega n_s}\int d^3v\Big[\Big(1 - \frac{kv_\parallel}{\omega}\Big)f_{so} - \frac{kv_\perp^2}{2\omega}\frac{\partial f_{s0}}{\partial v_\parallel}\Big]\frac{1}{kv_\parallel - \omega \pm \Omega_s},\end{aligned} \quad (3)$$

where the upper sign in Eq. (3) is a left circular polarization wave; and the lower sign in Eq. (3) is a right circular polarization wave, which corresponds to a whistler wave.

If the electron and ion velocity distribution can be approximately described by the bi-Maxwellian function with a field-aligned outflow drift velocity, their distribution function may be written as

$$f_{so}(v_\perp, v_\parallel) = n_s F_{\perp s}(v_\perp)F_{\parallel s}(v_\parallel), \quad (4)$$

where

$$F_{\perp s} = \frac{m_s}{2\pi T_{\perp s}}\exp\Big(-\frac{m_s v_\perp^2}{2T_{\perp s}}\Big),$$

$$F_{\parallel s} = \Big(\frac{m_s}{2\pi T_{\parallel s}}\Big)^{1/2}\exp\Big(-\frac{m_s(v_\parallel - V_{ds})^2}{2T_{\parallel s}}\Big), \quad (5)$$

where $T_{\parallel s}$ and $T_{\perp s}$ are the effective temperatures of electrons or ions ($s = e, i$) in the direction parallel and perpendicular magnetic field, respectively. V_{ds} is the field-aligned relative outflow drift velocity of electrons or ions. Substitute the distribution function Eq. (4) into dispersion equation (3). The integration over $v_\perp$ can be carried out by

$$\int_0^\infty v_\perp^2 F_{\perp s}(v_\perp)2\pi v_\perp dv_\perp = \frac{2T_\perp}{m_s}, \quad (6)$$

and the integration over $v_\parallel$ can be performed by

$$\begin{aligned}\int_{-\infty}^{\infty}\frac{F_{\parallel s}(v_\parallel)}{kv_\parallel - \Omega_s - \omega}dv_\parallel &= \frac{1}{k}\Big(\frac{m_s}{2T_{\parallel s}}\Big)^{1/2}Z(\zeta_s), \\ \int_{-\infty}^{\infty}\frac{k(v_\parallel - V_{ds})F_{\parallel s}(v_\parallel)}{kv_\parallel - \Omega_s - \omega}dv_\parallel &= 1 + \zeta_s Z(\zeta_s),\end{aligned} \quad (7)$$

where $\zeta_s = \frac{1}{k}\Big(\frac{m_s}{2T_{\parallel s}}\Big)^{1/2}(\omega - kV_{ds} + \Omega_s)$, usually $\zeta_s \gg 1$ in the region of whistler waves. $Z(x)$ is the dispersion function,[13] it can be approximately expressed for large argument x by

$$Z(x) = i\pi^{1/2}\exp(-x^2) - x^{-1}\Big[1 + \frac{1}{2x^2} + \frac{3}{4x^4} + - - - - - -\Big], \quad x \gg 1. \quad (8)$$

Using these formulas, the dispersion equation of whistler waves, propagating along the magnetic field, can be approximately written as

$$\mathrm{Re}\Lambda(k,\omega) \simeq 1 - \frac{k^2c^2}{\omega^2} - \sum_s \frac{\omega_{ps}^2}{\omega^2}\frac{\omega - kV_{ds}}{\omega - kV_{ds} + \Omega_s} = 0. \quad (9)$$

It is readily found that Eq. (9) is the dispersion relation of whistler waves in cold plasma, if the outflow drift velocity V_{ds} is negligible, and the anisotropy temperature ratio $\frac{T_{\perp s}}{T_{\parallel s}}$ only influences in the higher correction for the dispersion relation of whistler waves. The ion term in Eq. (9) is often neglected in the whistler frequency region of $\Omega_i \ll \omega \ll |\,\Omega_e\,|$. However, the electron outflow drift velocity V_{de} will influence the dispersion relation of whistler waves. The calculation results in the whistler frequency region $\Omega_i \ll \omega \ll |\,\Omega_e\,|$ with parameters of $\frac{\omega_{pe}}{|\,\Omega_e\,|} = 1$ and $V_{de} = 5\times V_A, 0, -5\times V_A$ (V_A is the local Alfvén velocity) have been shown in Fig. 2. It is found from Fig. 2 that, in the case of $V_{de} = 5\times V_A$, which corresponds to the electron outflow drift velocity parallel to the background magnetic field, the refractive index N of plasma is less than that without the electron outflow drift velocity. In contrast, the refractive index N increases in the case of $V_{de} = -5 \times V_A$, i.e. when the electron outflow drift velocity is anti-parallel to the background magnetic field.

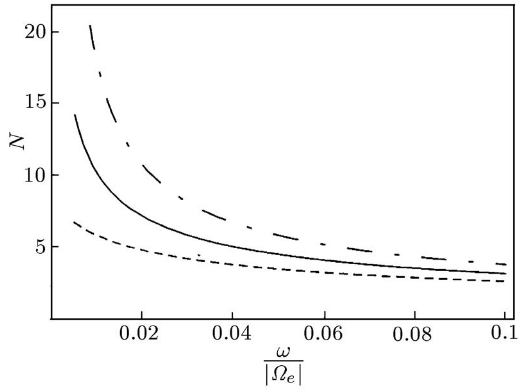

Fig. 2. Refractive index of plasma *versus* the whistler wave frequency (in units of electron cyclotron frequency) in the range of $\Omega_i \ll \omega \ll |\Omega_e|$ with the parameters $\frac{\omega_{pe}}{|\Omega_e|} = 1$ and $V_{de} = 0$ (solid line); $V_{de} = 5 \times V_A$ (dashed line); $V_{de} = -5 \times V_A$ (dot-dashed line), respectively.

The imaginary part of dispersion equation (3) can be expressed as

$$\mathrm{Im}\Lambda(k,\omega) = \sum_s \pi^{1/2} \frac{\omega_{ps}^2}{\omega^2} \frac{1}{k} \Big(\frac{m_s}{2T_{\parallel s}}\Big)^{1/2} \cdot \exp[-\zeta_s^2] \Big[(\omega - kV_{ds})\frac{T_{\perp s}}{T_{\parallel s}} + \Big(\frac{T_{\perp s}}{T_{\parallel s}} - 1\Big)\Omega_s\Big]. \quad (10)$$

On the other hand, the wave frequency can be written as $\omega = \omega_R + i\gamma$. It is assumed that the absolute value of the imaginary part frequency reads $|\gamma| \ll \omega_R$, then γ can be written as

$$\gamma = -\frac{\mathrm{Im}\Lambda(k,\omega_R)}{\frac{\partial \mathrm{Re}\Lambda(k,\omega_R)}{\partial \omega_R}}. \quad (11)$$

It is found from Eq. (9) that $\frac{\partial \mathrm{Re}\Lambda(k,\omega_R)}{\partial \omega_R} \simeq \frac{\omega_{pe}^2}{\omega_R^2 |\Omega_e|}$ in the whistler wave region of $\Omega_i \ll \omega \ll |\omega_e|$, which is always positive. Therefore, the whistler instability is not possible, or whistler waves may be excited, only if the condition of $\mathrm{Im}\Lambda(k,\omega_R) < 0$ is satisfied. $\mathrm{Im}\Lambda(k,\omega_R)$ in Eq. (10) consists of ion term and electron term. The ion term is often positive due to $\Omega_i > 0$. In other words, the ion term is often a stable factor for whistler instability. We can obtain an approximately lower threshold frequency ω_L of the whistler instability from the condition of $\mathrm{Im}\Lambda(k,\omega_L) = 0$:

$$\omega_L \Big(\frac{m_e}{m_i}\Big)^{1/2} \exp\Big(-\frac{m_i\omega_L^2}{2k^2T_{\parallel i}}\Big) \simeq \Big(\frac{T_{\perp e}}{T_{\parallel e}} - 1\Big) |\Omega_e| \exp\Big(-\frac{m_e\Omega_e^2}{2k^2T_{\parallel e}}\Big). \quad (12)$$

In the frequency region of $\omega > \omega_L$, the electron term should be much larger than the ion term; the ion term might be neglected in Eq. (10). In this case, the growth rate of whistler instability $\gamma > 0$ can be expressed approximately as

$$\gamma \simeq \pi^{1/2} \frac{|\Omega_e|}{k} \Big(\frac{m_e}{2T_{\parallel e}}\Big)^{1/2} \exp[-\zeta_e^2] \Big[\Big(\frac{T_{\perp e}}{T_{\parallel e}} - 1\Big) \cdot |\Omega_e| - (\omega - kV_{de})\frac{T_{\perp e}}{T_{\parallel e}}\Big], \quad (13)$$

where $\zeta_e = \frac{1}{k}\Big(\frac{m_e}{2T_{\parallel e}}\Big)^{1/2}(\omega - kV_{de} - |\Omega_e|)$. Thus, the whistler instability can be excited, if the electron anisotropy temperature ratio $\frac{T_{\perp e}}{T_{\parallel e}}$ satisfies

$$\frac{T_{\perp e}}{T_{\parallel e}} > 1 + \frac{\omega - kV_{de}}{|\Omega_e| + kV_{de} - \omega}. \quad (14)$$

From Eq. (14), we can also obtain an upper frequency threshold of whistler instability ω_H for a fixed electron anisotropy temperature ratio of $\frac{T_{\perp e}}{T_{\parallel e}}$,

$$\omega_H = \Big(1 - \frac{T_{\parallel e}}{T_{\perp e}}\Big) |\Omega_e| + kV_{de}. \quad (15)$$

If the electron outflow drift velocity V_{de} can be negligible, the electron anisotropy temperature ratio $\frac{T_{\parallel e}}{T_{\perp e}}$ and the upper frequency threshold ω_H are consistent with the previous results.[9,10]

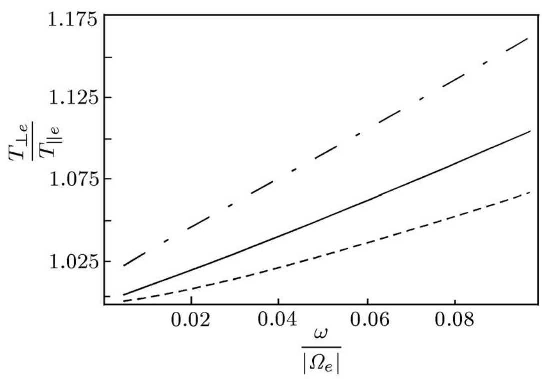

Fig. 3. Electron anisotropy temperature ratio $\frac{T_{\perp e}}{T_{\parallel e}}$ *versus* the whistler wave frequency (in units of electron cyclotron frequency) in the range of $\Omega_i \ll \omega \ll |\Omega_e|$ with the parameters the same as in Fig. 2.

However, it is known from the particle in cell simulation of fast magnetic reconnection that the electron outflow drift velocity V_{de} cannot be neglected; at somewhere during the magnetic reconnection,[5,6,14] the electron outflow drift velocity may reach a few of V_A. The calculation results of the electron anisotropy temperature ratio for the same parameters as in Fig. 2 are given in Fig. 3. From Eqs. (14) and (15) and Fig. 3, it is readily found that, compared to the case of $V_{de} = 0$, the electron outflow drift velocity makes the electron anisotropy temperature ratio decrease and the upper frequency threshold of whistler instability ω_H increase, when electron outflow drift velocity direction is parallel to the background magnetic field, as at points a and d in Fig. 1. Contrarily, the electron outflow drift velocity makes the electron anisotropy temperature ratio increase and the upper frequency threshold of whistler instability decrease, when the electron outflow drift velocity is anti-parallel to the background magnetic field, as at points b and c in Fig. 1. Therefore, the existence of electron outflow drift velocity will obviously change the threshold condition of whistler instability during the collisionless magnetic reconnection. Recently, Guo and Lu *et al.* have studied the whistler instability in magnetic reconnection by means of particle in cell simulation;[15] they find that not only is the electron velocity distribution anisotropic with a field-aligned outflow drift velocity, but also the whistler waves are excited in situ during the collisionless magnetic reconnection. This is consistent with the above theoretical model.

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