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Ion stochastic heating by obliquely propagating magnetosonic waves

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The ion motions in obliquely propagating Alfven waves with sufficiently large amplitudes have already been studied by Chen *et al*. [Phys. Plasmas **8**, 4713 (2001)], and it was found that the ion motions are stochastic when the wave frequency is at a fraction of the ion gyro-frequency. In this paper, with test particle simulations, we investigate the ion motions in obliquely propagating magnetosonic waves and find that the ion motions also become stochastic when the amplitude of the magnetosonic waves is sufficiently large due to the resonance at sub-cyclotron frequencies. Similar to the Alfven wave, the increase of the propagating angle, wave frequency, and the number of the wave modes can lower the stochastic threshold of the ion motions. However, because the magnetosonic waves become more and more compressive with the increase of the propagating angle, the decrease of the stochastic threshold with the increase of the propagating angle is more obvious in the magnetosonic waves than that in the Alfven waves. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4731707]

I. INTRODUCTION

Ion heating plays an important role in space and magnetically controlled laboratory plasmas, and wave-particle interactions are considered to be one of the most important mechanisms.[1–6] It is generally considered that the cyclotron resonant condition is necessary for wave-particle interactions.[7–11] The cyclotron resonant condition can be described as $\omega - k_{||}v_{||} = \pm n\Omega_0$ (where n is an integer, ω and $\mathbf{k}$ are the wave frequency and wave-vector, respectively, $\mathbf{v}$ is the particle velocity, and Ω_0 is the cyclotron frequency of the particle. The subscript "||" denotes the component parallel to the ambient magnetic field. + and – represent the left-hand and right-hand polarized waves, respectively), and the wave frequency is usually comparable to the cyclotron frequency. However, Chen *et al.*[12] proposed that an obliquely propagating low-frequency Alfven wave with sufficiently large amplitude can break the conservation of the magnetic moment at the frequency a fraction of the ion cyclotron frequency, and then the ions are stochastically heated by such a sub-cyclotron resonance. After applying Lie perturbation theory, Guo *et al.*[13] found that the ion stochastic heating occurs when the cyclotron resonances at sub-cyclotron frequencies start to overlap with their neighboring resonances. When a spectrum of Alfven waves is considered, the amplitude threshold of ion stochastic heating is found to be much lower than that of a monochromatic wave.[14,15] Evidences of ion heating by low-frequency Alfven waves have also been found in laboratory experiments.[16–18]

In this paper, we investigate ion stochastic heating by obliquely propagating magnetosonic waves. Compared with an Alfven wave, the magnetosonic wave is compressive, and the influences of such characteristics on ion stochastic heating are taken into account in this paper.

II. TEST PARTICLE SIMULATIONS

In this paper, we adopt test particle simulations to investigate ion stochastic heating by obliquely propagating magnetosonic waves. The plasma is uniformly magnetized with the background magnetic field, $\mathbf{B_0} = B_0\mathbf{i_z}$, and the magnetosonic waves with linear polarization propagate obliquely to the background magnetic field. In the laboratory frame X, Y, and Z, the magnetic and electric fields associated with the magnetosonic waves (which is derived from the ideal MHD equations[19] for cold plasma) can be modeled as

$$\mathbf{B_w} = \sum_{k=1}^{N} B_k \sin\psi_k \left(\mathbf{i_x} - \frac{k_x}{k_z}\mathbf{i_z}\right), \tag{1}$$

$$\mathbf{E_w} = -\sum_{k=1}^{N} v_A B_k \frac{k_t}{k_z} \sin\psi_k \mathbf{i_y}, \tag{2}$$

where $\psi_k = \omega t - (k_x X + k_z Z) + \varphi_k$, ω is the frequency of the wave, and φ_k is the random phase of mode k. $k_t = \sqrt{k_x^2 + k_z^2}$ is the wave number, and $k_x = k_t \sin\alpha$, $k_z = k_t\cos\alpha$ (where α is the propagating angle of the magnetosonic wave). N is the number of wave modes, and B_k is the wave amplitude of mode k. The dispersion relation of the magnetosonic wave is $\omega = k_t v_A$, and v_A is the Alfven speed.

In the wave frame $\mathbf{x} = \mathbf{X} - (v_A/\cos\alpha)t\mathbf{i}_z$, $\psi_k = k_x x + k_z z + \varphi_k$, and the wave electric field is eliminated. The magnetic field can be written as

$$\mathbf{B_w} = -\sum_{k=1}^{N} B_k \sin\psi_k \left(\mathbf{i_x} - \frac{k_x}{k_z}\mathbf{i_z}\right). \tag{3}$$

The ion motion is controlled only by the magnetic field, which can be described by the following equations:

$$m_i \frac{d\mathbf{v}}{dt} = q_i \mathbf{v} \times (\mathbf{B_0} + \mathbf{B_w}), \tag{4}$$

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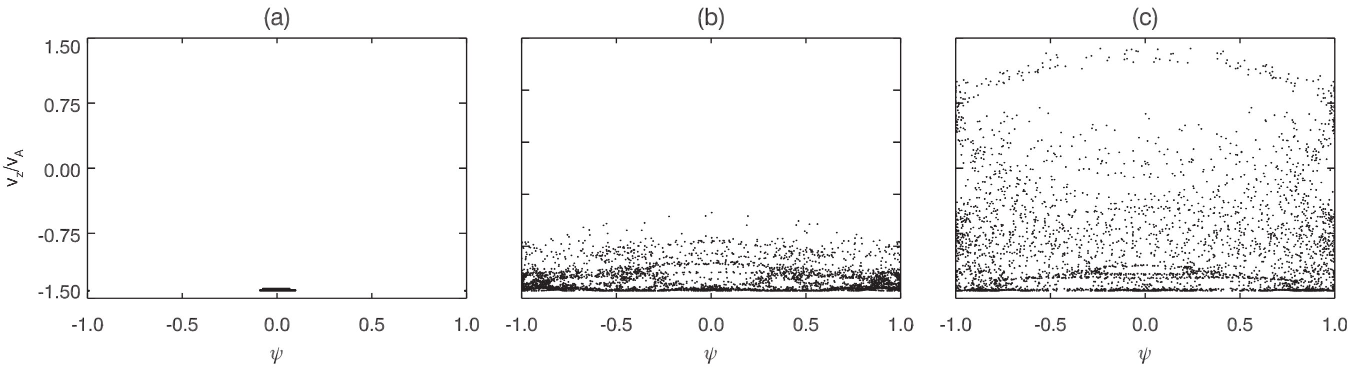

FIG. 1. Poincare plot for a monochromatic magnetosonic wave, and the parameters are $\omega = 0.25\Omega_i$, $\alpha = 45°$, and (a) $B_k^2/B_0^2 = 0.015$, (b) $B_k^2/B_0^2 = 0.079$, and (c) $B_k^2/B_0^2 = 0.085$.

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}, \tag{5}$$

where the subscript i denotes physical quantities associated with ion species i, and we consider proton in this paper. The equations are solved with Boris algorithm,[20] where the kinetic energy of the particle is conserved in the calculation of cyclotron motion. The time step is $\Omega_i \Delta t = 0.025$, where Ω_i is the proton cyclotron frequency.

III. SIMULATION RESULTS

In this paper, we investigate ion stochastic heating by obliquely propagating magnetosonic waves, and both a monochromatic wave and a spectrum of waves are considered. In a monochromatic magnetosonic wave, a Poincare plot of $\lambda = v_z/v_A$, $\psi = \cos(k_x x + k_z z + \varphi_k)$, formed by taking points when $v_y = 0$ and $\dot{v}_y > 0$ in a selected particle orbit, is constructed in the wave frame to study ion heating. Poincare plot is the intersection of an orbit in the state space of a continuous dynamical system with a certain lower dimensional subspace, called the Poincare section. Poincare plot preserves many properties of orbits of the original dynamical system and has a lower dimensional state space. Therefore, it is a useful tool to analyze the properties of a dynamical system.[21] In this paper, Figure 1 shows the Poincare plot of the ion motions for a monochromatic magnetosonic wave at different wave amplitude (a) $B_k^2/B_0^2 = 0.015$, (b) $B_k^2/B_0^2 = 0.079$, and (c) $B_k^2/B_0^2 = 0.085$, where $\omega = 0.25\Omega_i$ and $\alpha = 45°$. We assume that the initial ion distribution is cold in the laboratory frame. Therefore, initially the particle has the velocity $v_x = 0$, $v_y = 0$, and $v_z = -v_A/\cos\alpha$ in the wave frame. At $B_k^2/B_0^2 = 0.015$, the ion motions are quasi-periodic. With the increase of the wave amplitude, the ion motions become more and more stochastic due to the resonance with the wave at sub-cyclotron frequencies. The amplitude threshold is about $B_k^2/B_0^2 = 0.079$. At $B_k^2/B_0^2 = 0.085$, the ion velocity can readily diffuse from $v_z = -1.4v_A$ to about $v_z = 1.3v_A$, and the ion motions are obviously stochastic.

In Fig. 2, the effects of the wave propagating angle α on the ion motions are investigated. We keep $\omega = 0.25\Omega_i$, $\alpha = 60°$, and then plot the Poincare plot of the ion motions at different wave amplitude (a) $B_k^2/B_0^2 = 0.03$, (b) $B_k^2/B_0^2 = 0.036$, and (c) $B_k^2/B_0^2 = 0.045$. Similar to Fig. 1, the ion motions become stochastic with the increase of wave amplitude, and here the threshold is about $B_k^2/B_0^2 = 0.036$. We can find that the increase of the wave propagating angle can lower the threshold of the ion stochastic motions, which is the same as in the obliquely propagating Alfven wave.[12,14,15] Figure 3 shows the amplitude threshold of the ion stochastic motions at different frequency and propagating angle for both the monochromatic Alfven wave (solid lines) with a left-hand polarization and magnetosonic wave (dotted lines). The increase of either the wave frequency or the propagating angle can lower the amplitude threshold of the ion stochastic motions in both the magnetosonic wave and the Alfven

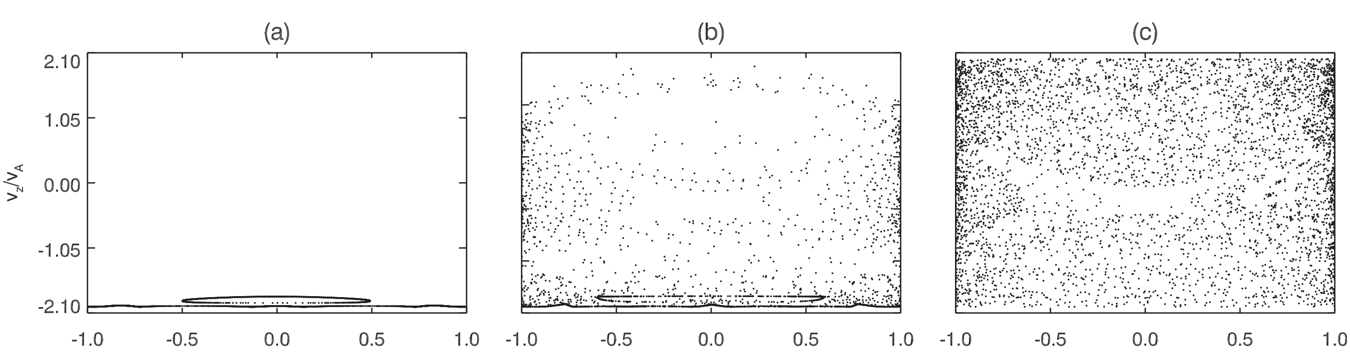

FIG. 2. Poincare plot for a monochromatic magnetosonic wave, and the parameters are $\omega = 0.25\Omega_i$, $\alpha = 60°$, and (a) $B_k^2/B_0^2 = 0.03$, (b) $B_k^2/B_0^2 = 0.036$, and (c) $B_k^2/B_0^2 = 0.045$.

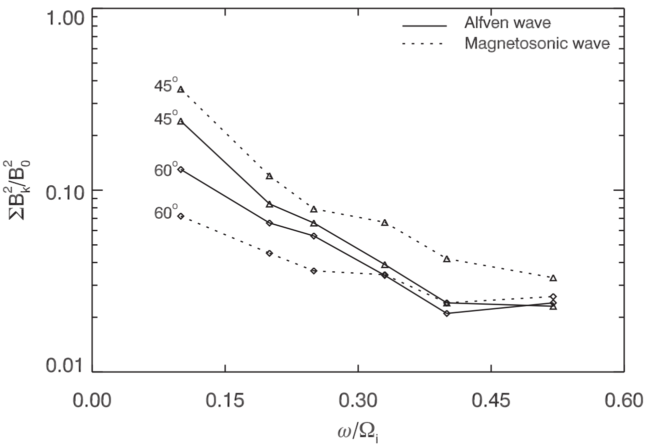

FIG. 3. The amplitude threshold of the ion stochastic motions at different frequency and propagating angle for both the monochromatic Alfven wave (solid lines) with a left-hand polarization and the magnetosonic wave (dotted lines).

wave. At $\alpha = 45^\circ$, the threshold of ion stochastic motions for the Alfven wave is lower than that for magnetosonic wave at the same frequency. However, at $\alpha = 60^\circ$, the threshold of ion stochastic motions for magnetosonic wave is lower than that for the Alfven wave. The reason for that is the magnetosonic wave becomes more and more compressive with the increase of the propagating angle.

Figures 4(a) and 4(b) show the evolution of the stochastic threshold at different propagating angle α and wave frequency for a monochromatic Alfven wave with a left-hand polarization and magnetosonic wave, respectively. At the same frequency, the stochastic thresholds for both the magnetosonic wave and Alfven wave decrease with the increase of the propagating angle. However, due to the compressibility of the magnetosonic wave, the decrease of its stochastic threshold with the increase of the propagating angle is more obvious than that of the Alfven wave.

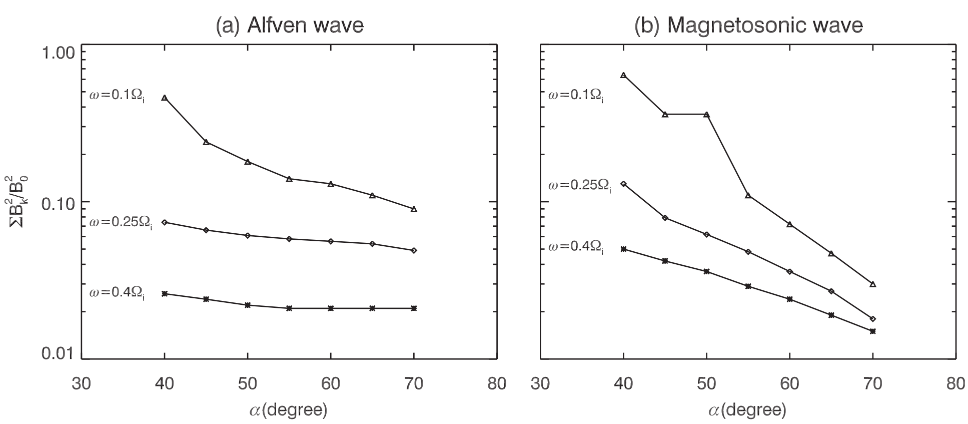

FIG. 4. The evolution of the stochastic threshold with different propagating angle α and wave frequency, (a) and (b) are for a monochromatic Alfven wave with a left-hand polarization and a magnetosonic wave, respectively.

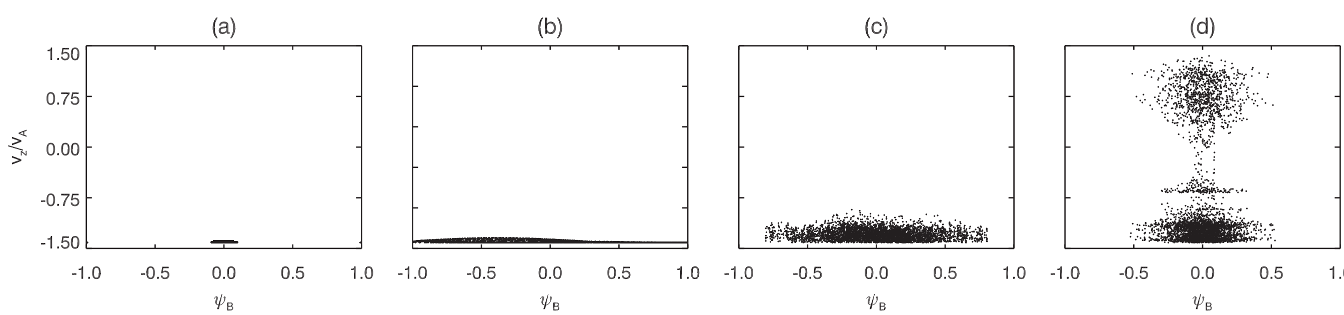

FIG. 5. Plot of $\lambda = v_z/v_A$, $\psi_B = (\sum_k B_k \cos(\psi_k))/\sum_k B_k$, by taking points when $v_y = 0$ and $\dot{v}_y > 0$ in the wave frame for (a) $N = 1$, (b) $N = 2$, (c) $N = 5$, and (d) $N = 21$. We keep $\alpha = 45^\circ$ and $\sum_k B_k^2/B_0^2 = 0.015$. The frequencies of the waves extend from $\omega_1 = 0.25\Omega_i$ to $\omega_N = 0.33\Omega_i$.

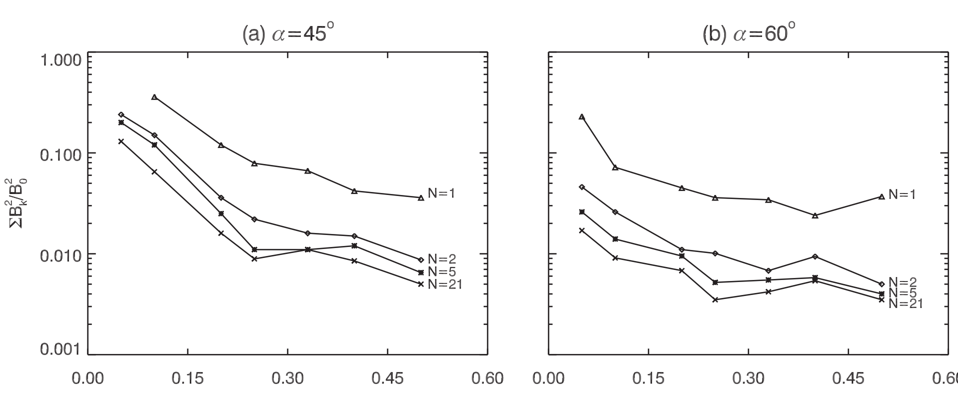

FIG. 6. The amplitude threshold of ion stochastic motions for the different number of wave modes and frequencies is (a) for $\alpha = 45^\circ$ and (b) for $\alpha = 60^\circ$. The range for the frequencies is kept as $\omega_N - \omega_1 = 0.08\Omega_i$.

We also study the effects of the number of wave modes on the ion motions. Here, $\alpha = 45^\circ$ and $\sum_k B_k^2/B_0^2 = 0.015$ are kept. The frequencies of the waves extend from $\omega_1 = 0.25\Omega_i$ to $\omega_N = 0.33\Omega_i$, and N is the number of wave modes. We set the frequencies of the wave modes as follows: $\omega_j = \omega_1 + (j-1)\Delta\omega$ $(j = 1, 2, \ldots, N)$, where $\Delta\omega = (\omega_N - \omega_1)/(N-1)$. The amplitude of individual wave modes satisfies the relation $(B_j/B_1)^2 = (\omega_j/\omega_1)^{-q}$, and q is set as 1.667. It means that the power frequency spectrum of the magnetosonic waves will have an index of -1.667. With a spectrum consisting of waves with different frequencies, a Poincare plot cannot be used to investigate the ion stochastic heating. However, we still can construct a plot of $\lambda = v_z/v_A$, $\psi_B = (\sum_k B_k \cos(\psi_k))/\sum_k B_k$, by taking points when $v_y = 0$ and $\dot{v}_y > 0$ in the wave frame.[15] Figure 5 shows such a plot for (a) $N=1$, (b) $N=2$, (c) $N=5$, and (d) $N=21$. With the increase of the wave number, the z component of the ion velocity can be diffused to a larger value. Therefore, the ion motions become stochastic. For example, at $N=21$, the ion can be diffuse from $v_z = -\sqrt{2}v_A$ to about $1.4v_A$, and its motions are obviously stochastic. The amplitude threshold of ion stochastic motions versus the wave frequency for different mode number is shown in Fig. 6, where (a) is for $\alpha = 45^\circ$ and (b) is for $\alpha = 60^\circ$. The width of spectrum of magnetosonic waves is kept as $\omega_N - \omega_1 = 0.08\Omega_i$. It is obvious that with the increase of the mode number, the threshold of ion stochastic motions decreases. We also can find that the threshold of ion stochastic motions will decrease as the wave propagating angle increases.

IV. CONCLUSIONS

We employ test particle simulations to investigate the ion motions in obliquely propagating magnetosonic waves. Similar to the Alfven wave, the obliquely propagating magnetosonic waves with sufficiently large amplitudes can also lead the ion motions to be stochastic due to resonances at sub-cyclotron frequencies, and then heat the ions.[12,14,15] With the increase of either the wave frequency or the propagating angle, the amplitude threshold of the ion stochastic motions will become lower. If a spectrum of magnetosonic waves is considered, the increase of number of wave modes can also lower the threshold of ion stochastic heating. However, because the magnetosonic waves become more and more compressive with the increase of the propagating angle, we can find that the decrease of the stochastic threshold with the increase of the propagating angle is more obvious in the magnetosonic waves than that in the Alfven waves.

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