The Efficiency of Ion Stochastic Heating by a Monochromatic Obliquely Propagating Low-Frequency Alfven Wave*

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Abstract The process of ion heating by a monochromatic obliquely propagating low-frequency Alfven wave is investigated. This process can be roughly divided into three stages: at first, the ions are picked up by the Alfven wave in several gyro-periods and a bulk velocity in the transverse direction is achieved; then, the ions are scattered in the transverse direction by the wave, which produces phase differences between the ions and leads to ion heating, especially in the perpendicular direction; and finally, the ions are stochastically heated due to the sub-cyclotron resonance. In this paper, with a test particle method, the efficiency and time scale of the ion stochastic heating by a monochromatic obliquely propagating low-frequency Alfven wave are studied. The results show that with the increase of the amplitude, frequency, and propagation angle of the Alfven wave, the efficiency of the ion stochastic heating increases, while the time scale of the ion stochastic heating decreases. With the increase of the plasma beta β, the ions are stochastically heated with less efficiency, and the time scale increases. We also investigate the heating of heavy ion species (He^{2+} and O^{5+}), which can be heated with a higher efficiency by the oblique Alfven wave.

Keywords: Alfven wave, stochastic heating, test particle method

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(Some figures may appear in colour only in the online journal)

1 Introduction

The plasma heating by Alfven waves is a fundamental physical process in space and magnetically controlled laboratory plasmas [1−4]. Most available works on this topic are focused on the resonant heating of the ions by Alfven waves [5−10], where the resonant condition $\omega - \mathbf{k} \cdot \mathbf{v} = n\Omega_i$ (ω and $\mathbf{k}$ are the frequency and wavevector of the Alfven waves, respectively, n is an integer and Ω_i is the ion gyro-frequency) between the ions and Alfven waves is necessary. There are still other mechanisms, such as linear mode coupling during turbulence cascade [11,12], which may also heat the plasma. Generally, in these mechanisms, the frequencies of the Alfven waves should be comparable to the ion gyro-frequency. Until now there has been no direct evidence showing the existence of such high frequency Alfven waves in the solar wind, and spacecraft observations have indicated that low-frequency Alfven waves with their frequency much smaller than the ion gyro-frequency exist prevalently in the solar corona and interplanetary space [13,14]. Therefore, ion heating by low-frequency Alfven waves is more attractive.

It is found that the pickup ions are able to be heated by parallel propagating low-frequency Alfven waves through a nonresonant process [15]. In such a process, the ions are at first picked up in the transverse direction by the wave and obtain a bulk transverse velocity. Then, the ions are heated due to the phase randomization between the ions. Chen et al. [16] further found that the ions can be stochastically heated through a sub-cyclotron resonance by an obliquely propagating low-frequency Alfven wave with a sufficiently large amplitude. When the spectrum of Alfven waves is considered, the amplitude threshold of ion stochastic heating can be much lower than that of a monochromatic wave [17]. Stochastic heating by oblique low-frequency Alfven waves may provide a possible way to heat plasma in the solar wind. However, before stochastic heating by oblique Alfven waves can be used in the solar wind, the efficiency and time scale of the heating should be evaluated. In this paper, with a test particle method, the efficiency and time scale of the ion stochastic heating by a monochromatic obliquely propagating low-frequency Alfven wave with different characteristics are investigated. The characteristics of the Alfven wave include the amplitude, propagating angle and frequency. Furthermore, we also consider the effects of the plasma

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beta and ion species on the ion heating.

2 Simulation model

In this paper, a test particle method is employed to investigate the ion heating by a monochromatic obliquely propagating low-frequency Alfven wave with the left-handed polarization. The plasma is uniformly magnetized with a background magnetic field, $\mathbf{B_0} = B_0\mathbf{i_z}$. The dispersion relation of the Alfven wave is $\omega = k_z V_A$, and V_A is the Alfven speed. In this paper, we investigate the particle motion in the wave frame. In such a frame, the wave electric field is eliminated, and the magnetic field can be written as

$$\mathbf{B}_w = B_w[-\cos(\alpha)\sin(\psi)\mathbf{i_x}+\cos(\psi)\mathbf{i_y}+\sin(\alpha)\sin(\psi)\mathbf{i_z}], \quad (1)$$

where $\psi = k_x x + k_z z$ is the wave phase, $\alpha = \arctan(k_x/k_z)$ is the propagating angle and B_w is the wave amplitude. The ion motion is controlled by the Lorentz force, which can be described by the following equations:

$$m_i \frac{d\mathbf{v}}{dt} = q_i \mathbf{v} \times (\mathbf{B}_0 + \mathbf{B}_w), \quad (2)$$

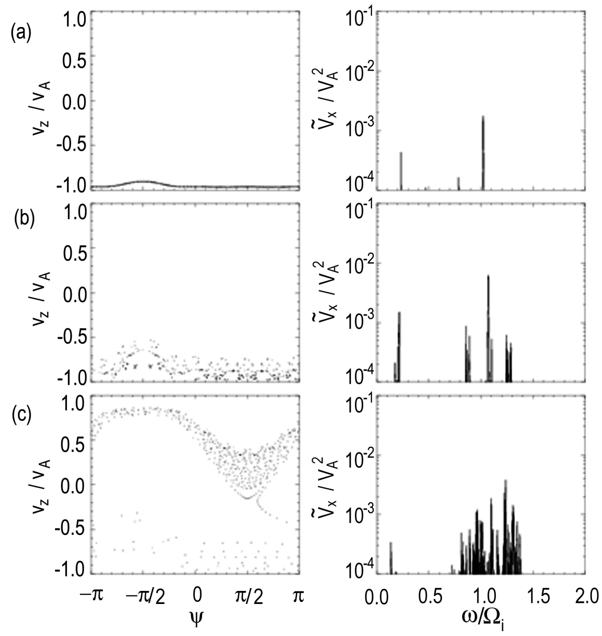
$$\frac{d\mathbf{r}}{dt} = \mathbf{v}, \quad (3)$$

where the subscript i denotes physical quantities associated with the ion species i. The equations are solved with the Boris algorithm [18], where the kinetic energy of the particle is conserved in the calculation.

In the following calculations, the initial velocity distribution function of the ions is drift-Maxwellian. The bulk velocity is $-V_A\mathbf{i_z}$. The particles move in the (x, z) plane, while their velocity components (v_x, v_y, v_z) are kept in the calculations. The number of grid cells is $n_z \times n_x = 48 \times 48$, and the size of grid cell is $\Delta z = \Delta x = 0.5\pi V_A \Omega_i^{-1}$ (where Ω_i is the ion cyclotron frequency). There are, on average, 100 macro-particles in every cell. The time step is $\Delta t = 0.025\Omega_i^{-1}$.

3 Simulation results

To investigate the ion behaviors in the monochromatic obliquely propagating Alfven wave, we can construct a Poincare plot of $(v_z/V_A, \psi)$ in the wave frame, formed by taking points when $v_y = 0$ and $\dot{v}_y > 0$ in a selected particle orbit. A Poincare plot is a useful tool to analyze the properties of a dynamical system, for it has a lower dimensional state space and preserves many properties of orbits of the original dynamical system. Fig. 1 illustrates the Poincare plots of the ion motions (left panels) and the corresponding power spectra of the ion velocity $v_x(t)$ (right panels) for (a) $B_w^2/B_0^2 = 0.02$, (b) $B_w^2/B_0^2 = 0.07$, (c) $B_w^2/B_0^2 = 0.16$. In the calculations, $\omega = 0.25$, $\alpha = 45°$, and $\beta = 0.01$. The power spectra of the ion velocity $v_x(t)$ is calculated by a fast fourier transform (FFT) of the time series of $v_x(t)$. When the wave amplitude is smaller, for example $B_w^2/B_0^2 = 0.02$, from the right panel, we can identify two main frequencies: one is at $0.25\Omega_i$, and the other is at $1.0\Omega_i$, which correspond to the Alfven wave and ion gyromotion. It indicates that the ion motion is quasi-periodical. With the increase of the wave amplitude, more and more discernable frequencies appear, and even the continuous spectrum can be formed when the amplitude is sufficiently large. The ion motions become stochastic due to the resonance at sub-cyclotron frequencies. The threshold for ion stochastic motion is about $B_w^2/B_0^2 = 0.07$. At $B_w^2/B_0^2 = 0.16$, the ion can easily diffuse from $v_z = -V_A$ to $0.9V_A$.



(a) $B_w^2/B_0^2 = 0.02$, (b) $B_w^2/B_0^2 = 0.07$, (c) $B_w^2/B_0^2 = 0.16$

Fig.1 Left panel: Poincare plots of the ion motions. Right panel: Power spectra of the ion velocity $v_x(t)$. The parameters are $\omega = 0.25\Omega_i$, $\alpha = 45°$, $\beta = 0.01$

Fig. 2 displays the scatter plots of the ions between z =0 and $60V_A\Omega_i^{-1}$ at (a) $\Omega_i t = 0$, (b) $\Omega_i t = 1$, (c) $\Omega_i t = 30$, (d) $\Omega_i t = 100$, and (e) $\Omega_i t = 2000$, respectively. The parameters are $\omega = 0.25\Omega_i$, $\alpha = 45°$, $B_w^2/B_0^2 = 0.08$, and $\beta = 0.01$. Initially, the ions satisfy the drift-Maxwellian distribution with the thermal velocity of $0.1V_A$. Obviously, the heating process has three steps. At $\Omega_i t = 1$, ions are rapidly picked up by the Alfven wave, and their transverse bulk velocity, induced by the Alfven wave, shows a simple sinusoid-like structure. However, there is no obvious heating at this time. Subsequently, due to the finite parallel thermal velocity of ions, part of the translational energy can be randomized into thermal energy through a non-resonant process [13]. As shown in Fig. 2(c), the ion heating occurs obviously in the perpendicular direction. Then, the ions are further stochastically heated in both directions, which is due to the resonance at sub-cyclotron frequencies.

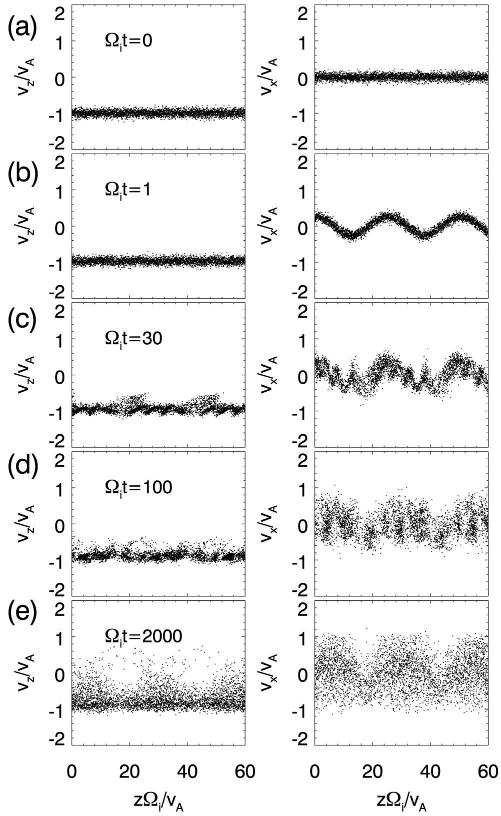


Fig.2 Scatter plots of the ions between 0 and $60v_{\rm A}\Omega_{\rm i}^{-1}$ at (a) $\Omega_{\rm i}t = 0$, (b) $\Omega_{\rm i}t = 1$, (c) $\Omega_{\rm i}t = 30$, (d) $\Omega_{\rm i}t = 100$, and (e) $\Omega_{\rm i}t = 2000$ respectively. The parameters are $\omega = 0.25\Omega_{\rm i}$, $\alpha = 45°$, $B_{\rm w}^2/B_0^2 = 0.08$, and $\beta = 0.01$

Fig. 3 displays the time evolution of ion distributions in the velocity space (converted into the laboratory frame). Here the ions in the adjacent four cells are recorded. The wave parameters are $\omega = 0.25\Omega_{\rm i}$, $\alpha = 45°$, $B_{\rm w}^2/B_0^2 = 0.08$, and $\beta = 0.01$, which can lead to the ion stochastic heating. Initially, the ions satisfy the Maxwellian distribution with the thermal velocity of $0.1V_{\rm A}$. At $\Omega_{\rm i}t = 30$, the ion distribution shows a shell-like structure in the v_x-v_y space due to the non-resonant hearing arising from phase randomization between the ions, and the ion heating is mainly in the perpendicular direction. Then, at $\Omega_{\rm i}t = 1000$ and 3000 the ions are further stochastically heated in both directions, which is due to the resonance at sub-cyclotron frequencies. Finally, the ion velocity distribution gradually becomes an isotropic distribution.

Fig. 4 shows the time evolution of the parallel and perpendicular temperatures, with the parameters of $B_{\rm w}^2/B_0^2$=0.08, ω=0.25, α=45°, and β=0.01. The ion temperatures in both directions were calculated with the following procedure: we first calculate $T_{||}$=$m_{\rm i}/k_{\rm B}\left\langle(v_z - \langle v_z\rangle)^2\right\rangle$, and $T_{\perp}$=$m_{\rm i}/2k_{\rm B}\left\langle(v_x - \langle v_x\rangle)^2 + (v_y - \langle v_y\rangle)^2\right\rangle$ in every cell (angle brackets denote an average over a cell), then the temperatures are averaged over all cells, so the contribution of the bulk velocity to the temperatures can be eliminated. Before $\Omega_i t$ =1000, the ions are nonresonantly heated due to the phase differences between the ions. Then, the ions are stochastically heated, and the heating rates ($\gamma_{\perp}$=$\dfrac{\Delta(T_{\perp}/T_0)}{\Delta(\Omega_{\rm i}t)}$ and $\gamma_{||}$=$\dfrac{\Delta(T_{||}/T_0)}{\Delta(\Omega_{\rm i}t)}$) can be calculated, which is about 0.0056 and 0.0061, respectively.

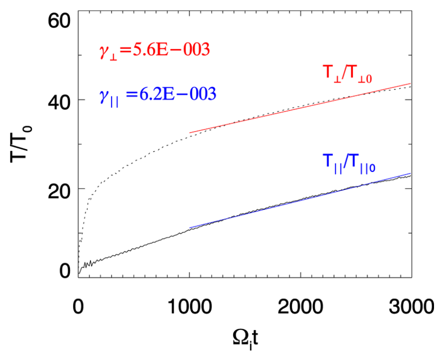


Fig.4 Time evolution of the parallel and perpendicular temperatures, and the parameters are $B_{\rm w}^2/B_0^2 = 0.08$, $\omega = 0.25\Omega_{\rm i}$, $\alpha = 45°$

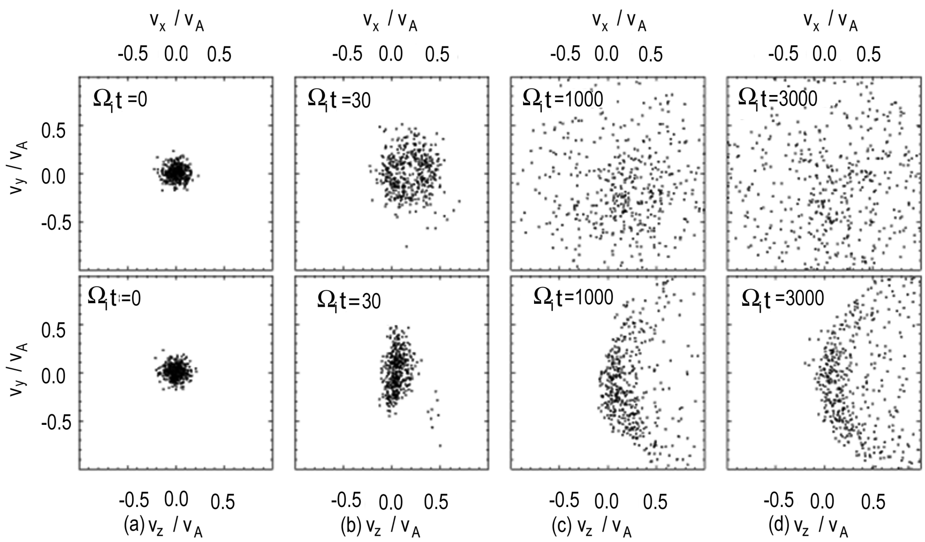


Fig.3 Scatter plots of the ions velocity in the velocity plane at (a) $\Omega_{\rm i}t = 0$, (b) $\Omega_{\rm i}t = 30$, (c) $\Omega_{\rm i}t = 1000$ and (d) $\Omega_{\rm i}t = 3000$, respectively. The wave parameters are $\omega = 0.25\Omega_{\rm i}$, $\alpha = 45°$, $B_{\rm w}^2/B_0^2 = 0.08$, and $\beta = 0.01$

Fig. 5 displays the time evolution of the perpendicular and parallel temperatures for different ion species, with the parameters of $B_{\rm w}^2/B_0^2 = 0.08$, $\omega = 0.25\Omega_{\rm i}$, $\alpha = 45°$, and $\beta = 0.01$. The heating of the heavy ions is more efficient than the heating of protons. The temperature of He^{2+} and O^{5+} tends to be isotropic, and the process of nonresonant heating due to phase randomization between ions is overlapped with the stochastic heating at sub-cyclotron frequencies.

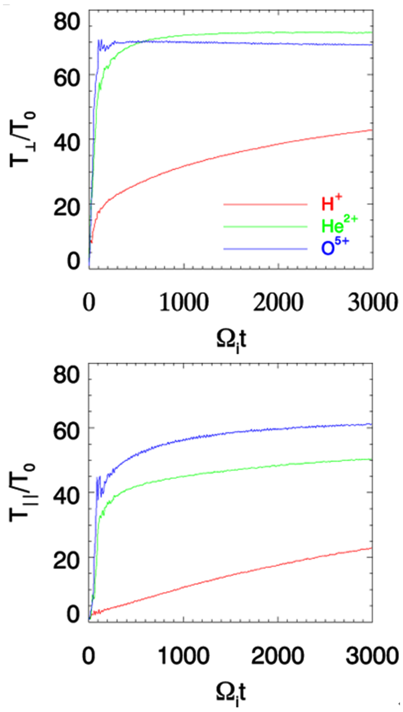


Fig.5 The time evolution of the perpendicular and parallel temperatures for different species of ions, in which the wave parameters are $B_{\rm w}^2/B_0^2 = 0.08$, $\omega = 0.25\Omega_{\rm i}$, $\alpha = 45°$, and $\beta = 0.01$

The dependences of the heating rates of ions ($\gamma_\perp$ and $\gamma_{||}$) due to the stochastic heating at sub-cyclotron frequencies on the wave amplitude $B_{\rm w}^2/B_0^2$, frequencies $\omega/\Omega_{\rm i}$, propagation angles $\tan\alpha$ and plasma beta β are also studied. The results are shown in Fig. 6. We can find that, with the increase of the wave amplitude, wave frequency and propagating angle, the rates of ion stochastic heating in both the parallel and perpendicular directions increase. The rates of ion heating are almost linearly proportional to the wave amplitude, wave frequency and $\tan\alpha$. With the increase of the plasma beta β, the rates of ion stochastic heating decrease. The parallel and perpendicular heating rates are almost the same in all cases, indicating that the stochastic heating is isotropic.

To study the time scale of the stochastic heating, we define the time scale $\tau = T_0 \Big/ \dfrac{\Delta T}{\Delta(\Omega_{\rm i} t)}$. Fig. 7 shows the dependences of the time scale of stochastic heating on the wave amplitude $B_{\rm w}^2/B_0^2$, frequencies $\omega/\Omega_{\rm i}$, propagation angles $\tan\alpha$ and plasma beta β. It is apparent that with the increase of the wave amplitude, wave frequency and propagating angle, the time scale of the ion stochastic heating in both the parallel and perpendicular decreases, while with the increase of the plasma beta β, the time scale of ion stochastic heating increases.

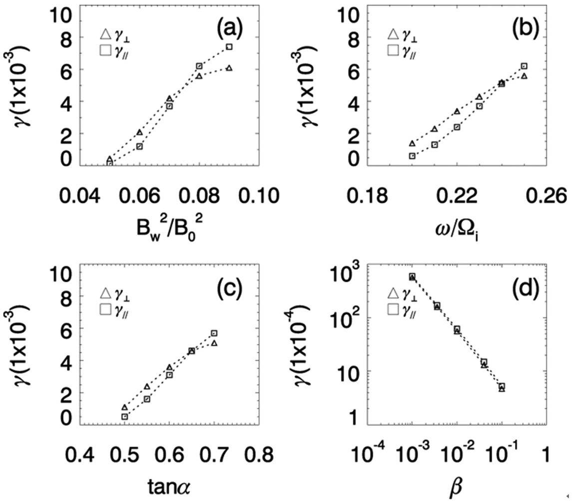


Fig.6 The stochastic heating rates in the parallel and perpendicular directions. The parameters are (a) $\omega = 0.25\Omega_{\rm i}$, $\alpha = 45°$, $\beta = 0.01$, (b) $(B_{\rm w}/B_0)^2 = 0.08$, $\alpha = 45°$, $\beta = 0.01$, (c) $(B_{\rm w}/B_0)^2 = 0.08$, $\omega = 0.33\Omega_{\rm i}$, $\beta = 0.01$, (d) $(B_{\rm w}/B_0)^2 = 0.08$, $\omega = 0.25\Omega_{\rm i}$, $\alpha = 45°$

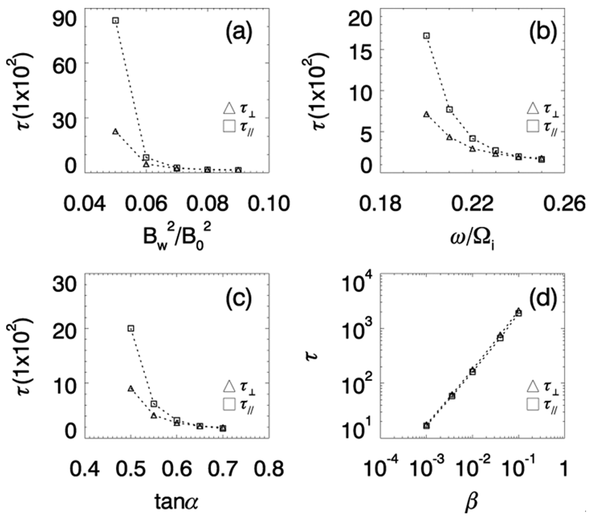


Fig.7 The time scale of stochastic heating in the parallel and perpendicular directions. The parameters are (a) $\omega = 0.25\Omega_{\rm i}$, $\alpha = 45°$, $\beta = 0.01$, (b) $(B_{\rm w}/B_0)^2 = 0.08$, $\alpha = 45°$, $\beta = 0.01$, (c) $(B_{\rm w}/B_0)^2 = 0.08$, $\omega = 0.33\Omega_{\rm i}$, $\beta = 0.01$, (d) $(B_{\rm i}/B_0)^2 = 0.08$, $\omega = 0.25\Omega_{\rm i}$, $\alpha = 45°$

4 Conclusions

In summary, with a test particle method, we have studied the ion heating by a monochromatic obliquely propagating low-frequency Alfven wave. Three stages are identified during the ion heating process: first, the ions are picked up by the Alfven wave within several gyro-periods; afterwards, the ions are heated rapidly through the nonresonant process due to phase difference between the ions, especially in the perpendicular direction; then the stochastic heating is dominant. The heating for heavy ions is more obvious than that for protons. For the stochastic heating of ions, the heating rates in both directions are almost the same. With the increase of amplitude, frequency, and propagation angle of the Alfven wave, the heating rates will also increase and the time scales for the heating decrease. With the increase of the plasma beta, the heating rates will decrease and the time scales for the heating increase. The obliquely propagating Alfven waves are considered to be generated from the perpendicular cascade and other mechanisms [19], and they exist pervasively in the solar corona and solar wind. In addition, the time scale of the ion stochastic heating is 10-20 min, which is comparable to the expanding time of the solar wind, therefore, our results should have some relevance to the plasma heating in the solar corona or solar wind. Of course, a test particle method doesn't consider the effects of the particles on the Alfven waves. When the stochastic heating of plasma by oblique Alfven waves is considered self-consistently, the Alfven waves will be dissipated and their amplitude will decrease. The efficiency of ion heating discussed in this paper will become small.

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