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- Roberts and Schulz bounce resonance diffusion coefficients are validated with simulations
- Bounce resonance diffusion coefficients are derived and validated for spatially confined waves
- Electron bounce resonance with magnetosonic waves is as important as gyroresonance

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Bounce resonance diffusion coefficients for spatially confined waves

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Abstract Theoretical bounce resonance diffusion coefficients from interactions between electrons and spatially confined waves are derived and validated. Roberts and Schulz bounce resonance diffusion coefficients assume waves to be present on the whole bounce trajectory of particles; therefore, they are not directly applicable to waves that have a finite spatial extent. We theoretically derive and numerically validate a new set of bounce resonance diffusion coefficients for spatially confined waves. We apply our analysis to magnetosonic waves, which are confined to equatorial regions, using a previously published magnetosonic wave model. We find that the bounce resonance diffusion coefficients are comparable to the gyroresonance diffusion coefficients. We conclude that bounce resonance diffusion with magnetosonic waves might play an important role in relativistic electron dynamics.

1. Introduction

Wave particle interactions play a very important role in the flux variation of relativistic electrons in Earth's outer radiation belt [see, e.g., *Thorne*, 2010]. Previous work mainly focused on gyroresonance, which violates the first and the second adiabatic invariants, and drift resonance, which violates the third adiabatic invariant. For example, gyroresonances with chorus [see, e.g., *Horne et al.*, 2005; *Thorne et al.*, 2013] and magnetosonic waves [*Horne et al.*, 2007; *Tao et al.*, 2009] have been shown to cause enhancement of MeV electron flux, while that with electromagnetic ion cyclotron (EMIC) waves [*Thorne and Kennel*, 1971; *Millan and Thorne*, 2007], chorus [see, e.g., *O'Brien et al.*, 2004], and plasmaspheric hiss [*Lyons and Thorne*, 1973; *Meredith et al.*, 2006] can contribute to the loss of radiation belt electrons. Drift resonance with ULF waves can lead to radial transport of electrons, resulting in energization or loss of radiation belt electrons [*Elkington et al.*, 2003; *Fei et al.*, 2006; *Shprits et al.*, 2006; *Hudson et al.*, 2014; *Dai et al.*, 2013, 2015].

While cyclotron and drift resonances have been studied in great detail, little attention has been paid to bounce resonance, which violates the second adiabatic invariant. *Roberts and Schulz* [1968] derived theoretical bounce resonance diffusion coefficients, assuming that waves are present on the whole trajectory of particles. They suggested that bounce resonance might lead to quick pitch angle scattering of near-equatorially trapped particles. However, the lack of knowledge of wave distributions made it difficult for the authors to accurately evaluate the importance of bounce resonance in their study. For example, observations show that magnetosonic waves are localized near geomagnetic equator within ~3° in latitude [*Russell et al.*, 1970; *Ma et al.*, 2013]; therefore, the theoretical diffusion coefficients of *Roberts and Schulz* [1968] cannot be directly used to evaluate the importance of bounce resonance with magnetosonic waves. Recently, *Shprits* [2009] surveyed potential waves that can bounce resonance with radiation belt electrons, suggesting that bounce resonances with magnetosonic waves and EMIC waves may lead to pitch angle scattering of near-equatorially mirroring particles and they may also result in local acceleration of electrons. However, no quantitative estimates of the bounce resonance effects have been made by *Shprits* [2009].

The purpose of this work is to derive the bounce resonance diffusion coefficients for spatially confined waves like magnetosonic waves and provide the first quantitative calculation of the bounce resonance diffusion coefficients of magnetosonic waves. Like *Horne et al.* [2007], we do not consider fine structures of magnetosonic waves [*Fu et al.*, 2014; *Boardsen et al.*, 2014] and assume that the wave spectrum is broad enough to allow the use of a diffusion approach to describe the particle dynamics. Also, we only consider bounce resonance, therefore excluding nonresonant interactions like transit time scattering [*Bortnik and Thorne*, 2010; *Bortnik et al.*, 2015; *Li et al.*, 2014]. We start from a review of the derivation of the *Roberts and Schulz* [1968] theoretical bounce resonance diffusion coefficients in section 2.1, using the approach given by *Schulz and Lanzerotti* [1974]. Then we derive, in section 2.2, a new set of bounce resonance diffusion coefficients for spatially confined waves. In section 3, we use guiding center test particle simulations to validate both sets of theoretical diffusion coefficients and evaluate the importance of bounce resonance diffusion by magnetosonic waves in radiation belt electron dynamics. We then summarize our work and discuss possible implications of the study in section 4.

2. Theoretical Bounce Resonance Diffusion Coefficients

2.1. The Roberts and Schulz Diffusion Coefficients

A set of theoretical bounce resonance diffusion coefficients have been derived by *Roberts and Schulz* [1968] and *Schulz and Lanzerotti* [1974, pp. 62–65]. In this section, we give a brief review of the derivation of the diffusion coefficients given by *Schulz and Lanzerotti* [1974]. The bounce resonance violates the second adiabatic invariant while preserving the first invariant, therefore allowing the use of guiding center equations of motion. For a dipole geomagnetic field, the guiding center equations can be simplified as

$$\frac{\mathrm{d}p_\parallel}{\mathrm{d}t} + \frac{M}{\gamma}\frac{\partial B}{\partial s} = f_\parallel(s,t), \tag{1}$$

where s is the distance from the equatorial plane along a field line, γ the Lorentz factor, $f_\parallel(s,t)$ the perturbation force parallel to the background magnetic field, and $M = p_\perp^2/2mB$ the first adiabatic invariant. Here m is the particle's rest mass, and $p_\parallel$ and $p_\perp$ are the particle's momentum parallel and perpendicular to the background geomagnetic field B, respectively. Multiplying equation (1) by $p_\parallel/m$ and using that $p_\parallel = \gamma m\dot{s}$, equation (1) can be written as

$$\mathrm{d}W/\mathrm{d}t = (p_\parallel/m)f_\parallel(s,t), \tag{2}$$

with $W = (p_\parallel^2/2m) + MB$. Following *Schulz and Lanzerotti* [1974], for a time interval $0 < t < \tau$, the oscillatory force is represented using Fourier series as

$$f_\parallel(s,t) = \sum_{n=1}^{\infty} f_n \cos(k_{\parallel n}s - \omega_n t + \phi_n), \tag{3}$$

where $\omega_n = 2\pi n/\tau$, $k_{\parallel n}$ is the parallel wave vector, and ϕ_n is the initial phase of the nth component. According to *Schulz and Lanzerotti* [1974], the contribution of each component to the mean square force perturbation $\langle [f_\parallel(s,t)]^2\rangle$ is $(1/2)f_n^2$, which resides in a frequency interval $\Delta\omega = 2\pi/\tau$, thus, the spectral density at ω_n is $\mathcal{F}(\omega_n) = (\tau/4\pi)f_n^2$.

To obtain bounce resonance diffusion coefficients, we integrate equation (2) along the unperturbed orbit of a particle. For particles mirroring close to the magnetic equator, the unperturbed trajectory is approximated by

$$s(t) \approx s_0 \sin(\Omega_b t + \theta_0), \tag{4}$$

where θ_0 is the initial bounce phase, and $s_0 = p_{\parallel e}/(\gamma m\Omega_b)$, with $p_{\parallel e}$ the parallel momentum at the geomagnetic equator and Ω_b the bounce frequency. Following *Schulz and Lanzerotti* [1974], we choose $\tau = NT_b$, where N is a large number and T_b is the bounce period; therefore, $\omega_n = (n/N)\Omega_b$. From equations (2)–(4), the change of W at $t = \tau$ is

$$\Delta W = \sum_{n=1}^{\infty} (p_{\parallel e}/m\Omega_b) f_n \Delta W_n, \tag{5}$$

where

$$\Delta W_n = \int_{\theta_0}^{2\pi N+\theta_0} \cos\theta \cos\left(k_{\parallel n}s_0 \sin\theta - \frac{n}{N}\theta + \frac{n}{N}\theta_0 + \phi_n\right)\mathrm{d}\theta. \tag{6}$$

Here $\theta = \Omega_b t + \theta_0$ is the particle's bounce phase. Using $\exp(iz\sin\theta) = \sum_{l=-\infty}^{\infty} J_l(z)\exp(il\theta)$, equation (6) is

$$\Delta W_n = \frac{1}{2}\sum_{l=-\infty}^{\infty} J_l(z_n)\int_{\theta_0}^{2\pi N+\theta_0}\left[f_+(\theta)+f_-(\theta)\right]\mathrm{d}\theta, \tag{7}$$

with $z_n = k_{\parallel n}s_0$, $f_\pm(\theta) = \cos[l_\pm\theta + (n/N)\theta_0 + \phi_n]$, and $l_\pm = l - n/N \pm 1$.

In this work, we consider only bounce resonance; i.e., $\omega_n/\Omega_b = n/N = l_0$, and l_0 is an integer. Equation (7) is then reduced to

$$\Delta W_{l_0N} = \Omega_b\tau l_0\frac{J_{l_0}(z_{l_0N})}{z_{l_0N}}\cos(l_0\theta_0 + \phi_{l_0N}). \tag{8}$$

It is straightforward to show that the contribution of nonresonant terms to ΔW_n in equation (7) vanishes. Substituting equation (8) into equation (5) gives

$$\Delta W = \sum_{l_0=1}^{\infty}\frac{p_{\parallel e}\tau}{m}f_{l_0N}l_0\frac{J_{l_0}(z_{l_0N})}{z_{l_0N}}\cos(l_0\theta_0 + \phi_{l_0N}). \tag{9}$$

The bounce resonance diffusion coefficient D_{WW} is defined by $D_{WW} = \langle(\Delta W)^2\rangle/2\tau$, where $\langle\cdot\cdot\rangle$ denotes averaging over θ_0 and ϕ_n. Using equation (9), it is straightforward to obtain

$$D_{WW} = \pi\sum_{l_0=1}^{\infty}\left[\frac{p_{\parallel e}}{m}l_0J_{l_0}(z_{l_0N})/z_{l_0N}\right]^2\mathcal{F}(l_0\Omega_b). \tag{10}$$

Here $z_{l_0N} = k_{\parallel l_0N}s_0$, with $k_{\parallel l_0N}$ the parallel wave vector corresponding to $\omega_{l_0N} = l_0\Omega_b$.

The bounce resonance diffusion coefficient given in equation (10) can be shown to be consistent with that given by *Roberts and Schulz* [1968], which is

$$D_{WW} = \frac{\gamma^2\Omega_b^2}{2\pi}\sum_{l_0=1}^{\infty}l_0^2\int_{-\infty}^{\infty}\mathrm{d}k_\parallel\mathcal{K}_F(k_\parallel, l_0\Omega_b)\frac{J_{l_0}^2(k_\parallel s_0)}{k_\parallel^2}, \tag{11}$$

where the function $\mathcal{K}_F(k_\parallel, l_0\Omega_b)$ satisfies

$$\frac{1}{2\pi^2}\int_{-\infty}^{\infty}\mathrm{d}k_\parallel\mathcal{K}_F(k_\parallel, l_0\Omega_b) = \mathcal{F}(l_0\Omega_b). \tag{12}$$

Letting the $k_\parallel$ dependence to be $\delta(k_\parallel - k_{\parallel l_0N})$ in $\mathcal{K}_F(k_\parallel, l_0\Omega_b)$, equation (11) is reduced to equation (10).

2.2. Bounce Resonance Diffusion Coefficients for Spatially Confined Waves

The Roberts and Schulz bounce resonance diffusion coefficients (equation (10) or (11)) assume that the wave field covers the whole bounce trajectory. On the other hand, realistic waves generally only cover part of the bounce trajectory of a particle. For example, *Horne et al.* [2007] used a magnetosonic wave model where the waves are confined to a magnetic latitude range of $|\lambda| \le 3°$. Except at very large equatorial pitch angles ($\alpha_0 \gtrsim 84°$), particles can bounce out of this latitude range. Therefore, the Roberts and Schulz diffusion coefficients are not directly applicable to spatially confined waves like magnetosonic waves.

In this section, we follow the approach of *Schulz and Lanzerotti* [1974] and derive the bounce resonance diffusion coefficients for spatially confined waves. As a typically example of spatially confined waves, *Horne et al.* [2007] assumed that the magnetosonic wave power is constant within $|\lambda| \le \lambda_0$ with λ_0=3° and is zero outside this latitude range. Following *Horne et al.* [2007], we assume that the perturbation force for spatially confined waves is represented by

$$f_\parallel(s,t) = \begin{cases}\sum_{n=1}^{\infty} f_n\cos(k_{\parallel n}s - \omega_n t + \phi_n) & |s| < s_{\max},\\ 0 & |s| \ge s_{\max},\end{cases} \tag{13}$$

where $s_{\max}$ denotes the distance from the equatorial plane to the boundary of magnetosonic waves along a field line. Note here that, for simplicity, we do not consider a spread in wave normal angle for a given frequency.

From $\mathrm{d}s = LR_E(1 + 3\sin^2\lambda)^{1/2}\cos\lambda\mathrm{d}\lambda$, L being the L shell value and R_E is Earth radius, it is straightforward to get $s_{\max}$ from $s_{\max} = \int_0^{\lambda_0}(\mathrm{d}s/\mathrm{d}\lambda)\mathrm{d}\lambda$ and

$$s_{\max} = \frac{1}{2}\left[\sin\lambda_0\sqrt{3\sin^2\lambda_0 + 1} + \ln\left(\sqrt{3\sin^2\lambda_0 + 1} + \sqrt{3}\sin\lambda_0\right)/\sqrt{3}\right]LR_E. \tag{14}$$

Approximating the unperturbed bounce motion as $s(t) \approx s_0\sin\theta$, then the oscillatory force is nonzero only when the bounce phase is within the range $\sum_{r=0}^{\infty}(r\pi - \theta_{\max}, r\pi + \theta_{\max})$, where $\theta_{\max}$ is

$$\theta_{\max} = \begin{cases} \arcsin(s_{\max}/s_0) & s_{\max} < s_0, \\ \pi/2 & s_{\max} \geq s_0. \end{cases} \tag{15}$$

Considering that the integrand of equation (6) has a period of $2\pi N$, the integration range in equation (6) becomes

$$\int_0^{\theta_{\max}}\mathrm{d}\theta + \sum_{r=1}^{2N-1}\int_{r\pi-\theta_{\max}}^{r\pi+\theta_{\max}}\mathrm{d}\theta + \int_{2N\pi-\theta_{\max}}^{2N\pi}\mathrm{d}\theta, \tag{16}$$

which can be combined to give

$$\sum_{r=0}^{r=2N-1}\int_{r\pi-\theta_{\max}}^{r\pi+\theta_{\max}}\mathrm{d}\theta. \tag{17}$$

Considering only bounce-resonant terms; i.e., $n/N = l_0$ with l_0 an integer, we find

$$\Delta W = \frac{p_{\parallel e}\tau}{m}\sum_{l_0=1}^{\infty}\cos(l_0\theta_0 + \psi_{l_0N})\frac{f_{l_0N}}{\pi}(\Delta W_{l_01} + \Delta W_{l_02}), \tag{18}$$

where

$$\Delta W_{l_01} = \frac{l_0}{z_{l_0N}}J_{l_0}(z_{l_0N})[2\theta_{\max} + \sin(2\theta_{\max})], \tag{19}$$

$$\Delta W_{l_02} = \sum_{\substack{|l-l_0|\neq 1 \\ l-l_0=\mathrm{odd}}}\left(\frac{\sin l_+\theta_{\max}}{l_+} + \frac{\sin l_-\theta_{\max}}{l_-}\right)J_l(z_{l_0N}). \tag{20}$$

The bounce resonance diffusion coefficient D_{WW} for spatially confined waves is therefore

$$D_{WW} = \sum_{l_0=1}^{\infty}\left[\frac{p_{\parallel e}}{m}(\Delta W_{l_01} + \Delta W_{l_02})\right]^2\mathcal{F}(\omega_{l_0N})/\pi, \tag{21}$$

where $\omega_{l_0N} = l_0\Omega_b$. It is straightforward to see that equation (21) is the same as equation (10) if $\theta_{\max} = \pi/2$. For electromagnetic perturbations like magnetosonic waves, the parallel force $f_\parallel(s,t)$ is

$$f_\parallel(s,t) = -\frac{M}{\gamma}\frac{\partial B_\parallel(s,t)}{\partial s} + qE_\parallel(s,t). \tag{22}$$

Therefore, the spectral density of the force in equation 21 is related to the spectral densities of the magnetic and electric fields by [*Schulz and Lanzerotti*, 1974]

$$\mathcal{F}(\omega) = \frac{M^2}{\gamma^2}k_\parallel^2\mathcal{B}_\parallel(\omega) + q^2\mathcal{E}_\parallel(\omega), \tag{23}$$

where $\mathcal{B}_\parallel$ and $\mathcal{E}_\parallel$ are the spectral density of the parallel wave magnetic field and electric field, respectively.

The bounce resonance diffusion coefficient D_{WW} can be converted to equatorial pitch angle and energy diffusion coefficients that are commonly used in global radiation belt modeling. The particle's equatorial pitch angle α_0 satisfies $\cot^2 \alpha_0 = W/MB_e - 1$, where B_e is the geomagnetic field strength at the magnetic equator; therefore,

$$\frac{\partial \alpha_0}{\partial W} = -\frac{\sin^2 \alpha_0 \tan \alpha_0}{2MB_e}. \tag{24}$$

Similarly, since $E = (\gamma - 1)mc^2$ and $\gamma^2 = 1 + 2W/mc^2$, we have

$$\frac{\partial E}{\partial W} = \frac{1}{\sqrt{1 + 2W/mc^2}} = \gamma^{-1}. \tag{25}$$

Correspondingly, the equatorial pitch angle and energy diffusion coefficients are

$$D_{\alpha_0\alpha_0} = D_{WW}\left(\frac{\partial \alpha_0}{\partial W}\right)^2, \quad D_{\alpha_0 E} = D_{WW}\frac{\partial \alpha_0}{\partial W}\frac{\partial E}{\partial W}, \text{ and } D_{EE} = D_{WW}\left(\frac{\partial E}{\partial W}\right)^2. \tag{26}$$

Note that for bounce resonance, the cross diffusion coefficient $D_{\alpha_0 E}$ is always negative for $0 < \alpha_0 < \pi/2$. This is because the conservation of $M = p_{\perp e}^2/2mB_e$ requires that an increase in the equatorial pitch angle always leads to a decrease in energy.

3. Simulation Results

3.1. Validation of the Roberts and Schulz Diffusion Coefficients

In this section, we will use test particle simulations to validate the Roberts and Schulz diffusion coefficients (equations (10) and (26)) [*Schulz and Lanzerotti*, 1974; *Roberts and Schulz*, 1968] using magnetosonic waves. For simplicity, we use a simplified dipole geomagnetic field with $B_z(\lambda) = B_e\sqrt{1 + 3\sin^2\lambda}/\cos^6\lambda$ and $B_e \approx 0.31/L^3$ Gauss. Following *Tao et al.* [2012], we choose $B_x = -x(\mathrm{d}B_z/\mathrm{d}z)/2$ and $B_y = -y(\mathrm{d}B_z/\mathrm{d}z)/2$ in order to satisfy $\nabla \cdot \boldsymbol{B} = 0$. We consider a magnetosonic wave field with oblique propagation in x-z plane; i.e.,

$$\boldsymbol{E} = \sum_{i=1}^{N} \boldsymbol{e}_x E_{xi}\sin\phi_i + \boldsymbol{e}_y E_{yi}\cos\phi_i + \boldsymbol{e}_z E_{zi}\sin\phi_i, \tag{27}$$

$$\boldsymbol{B} = \sum_{i=1}^{N} \boldsymbol{e}_x B_{xi}\cos\phi_i + \boldsymbol{e}_y B_{yi}\sin\phi_i + \boldsymbol{e}_z B_{zi}\cos\phi_i. \tag{28}$$

Here $\phi_i = \int \boldsymbol{k}_i \cdot \mathrm{d}\boldsymbol{r} - \omega_i t + \phi_{i0}$ is the wave phase of the ith component, with ω_i the wave frequency, ϕ_{i0} the wave initial phase, $\boldsymbol{k}_i$ the wave vector given by $\boldsymbol{k}_i = k_i(\sin\psi_i, 0, \cos\psi_i)$, where ψ_i is the wave normal angle. The amplitudes of the electric and magnetic fields are related by cold plasma dispersion relation as in *Tao and Bortnik* [2010].

The magnetosonic wave model used in the simulation is adopted from *Horne et al.* [2007]. The magnetic field spectral density is $\mathcal{B}(\omega) \propto \exp(-(\omega - \omega_m)^2/\delta\omega^2)$, $\omega_m/|\Omega_e| = 3.49 \times 10^{-3}$, $\delta\omega/|\Omega_e| = 8.86 \times 10^{-4}$, and a wave amplitude $B_w = 218$ pT in the range $0.0026|\Omega_e| < \omega < 0.0044|\Omega_e|$, where Ω_e is the equatorial electron cyclotron frequency. The field amplitude of the ith component is determined correspondingly using the method in *Tao et al.* [2012]. For simplicity, we set the wave normal angle $\psi = 89°$ instead of a Gaussian distribution in $\tan\psi$ as in *Horne et al.* [2007]. Furthermore, to be consistent with the assumption of *Roberts and Schulz* [1968], we use an unrealistic wave model in this part by assuming that the wave field is present at all latitudes and spectral density $\mathcal{B}$ remains constant. On the other hand, the wave field is present only near the equator in *Horne et al.* [2007]. Finally, the plasma density is assumed to be $f_{\mathrm{pe}}/f_{\mathrm{ce}} = 3$, independent of latitude.

We use the guiding center equations in *Tao et al.* [2007] to trace test particles. A fourth-order Runge-Kutta method is used to solve the set of guiding center equations. To calculate the diffusion coefficients for a given α_0 and E, the initial equatorial pitch angle and energy of all test particles are set to be α_0 and E, respectively. To create ensemble of particles with different initial bounce phases and initial wave phases, we perform 20 batches of simulations for a given α_0 and E. In each batch, we use 100 particles whose initial bounce phase is randomly chosen from $[0, 2\pi]$. Similarly, the wave initial phase is randomly distributed in $[0, 2\pi]$. From batch to batch, we use a different set of random initial wave phases and initial particle bounce phases. Sample trajectories of five particles from a simulation are shown in Figure 1 (left), which demonstrates that the

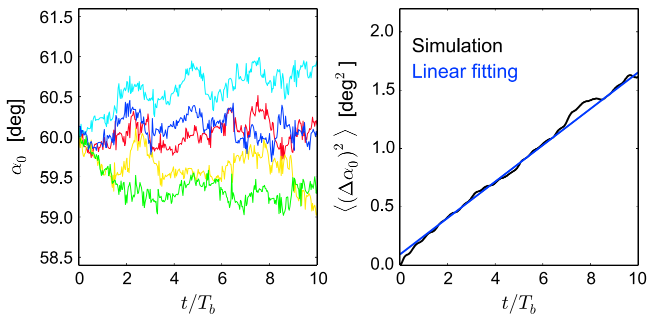

Figure 1. (left) Changes of α_0 of five randomly selected electrons, represented by different colors. (right) The evolution of $\langle(\Delta\alpha_0)^2\rangle$ with time for initial α_0=60°. The blue line is the corresponding linear fitting.

motion of particles are indeed stochastic from bounce resonance with a broadband magnetosonic wave field. The test particle diffusion coefficients are obtained from $D_{\alpha_0\alpha_0} = \langle(\Delta\alpha_0)^2\rangle/2\tau$ and $D_{EE} = \langle(\Delta E)^2\rangle/2\tau$, where $\Delta\alpha_0 = \alpha_0 - \langle\alpha_0\rangle$ and $\Delta E = E - \langle E\rangle$. Here $\langle\ldots\rangle$ means averaging over all particles in a simulation. We then perform a linear fitting to $\langle(\Delta\alpha_0)^2\rangle$ (or $\langle(\Delta E)^2\rangle$ for D_{EE}) as a function of time, and the test particle diffusion coefficient is then half of the slope. This process is illustrated in Figure 1 (right) for $D_{\alpha_0\alpha_0}$.

The comparison between the numerical diffusion coefficients and Roberts and Schulz diffusion coefficients is shown in Figure 2 for E = 1 MeV electrons at L = 4.5. We compare both $D_{\alpha_0\alpha_0}$ and D_{EE}/E^2. This comparison demonstrates that the theoretical diffusion coefficients of Roberts and Schulz agree well with test particle simulations, which proves the correctness of the Roberts and Schulz bounce resonance diffusion coefficients when the assumptions of the theory are satisfied.

3.2. Validation of the Diffusion Coefficients for Spatially Confined Waves

We now validate the diffusion coefficients for spatially confined waves using magnetosonic waves. The setup of the wave field is the same as in the section 3.1, except that we now consider realistic magnetosonic waves; i.e., the wave field is present only up to $|\lambda| \leq 3°$. Following *Horne et al.* [2007], the spectral density $\mathcal{B}(\omega)$ remains constant within $|\lambda| \leq 3°$ and is zero beyond this latitude range. Other parameters of simulations are exactly the same as in section 3.1. The resulting comparison of theoretical diffusion coefficients (equations (21)–(26)) with test particle simulations are shown in Figure 3. The good agreement between theory and simulation proves that the newly derived theoretical diffusion coefficients for spatially confined waves are correct.

Since we have validated the theoretical diffusion coefficients for spatially confined waves, we use them to estimate the importance of bounce resonance diffusion in radiation electron dynamics. Following *Horne et al.* [2007], we calculate the theoretical $D_{\alpha_0\alpha_0}$ and D_{EE}/E^2 for different equatorial pitch angles and energies at two different f_{pe}'s, shown in Figure 4. For energy E=1 MeV, the maximum $D_{\alpha_0\alpha_0}$ and D_{EE}/E^2 from bounce resonance are close to 10^{-5} at f_{pe}/f_{ce}=3, comparable to that from gyroresonance [*Horne et al.*, 2007, Figure 4].

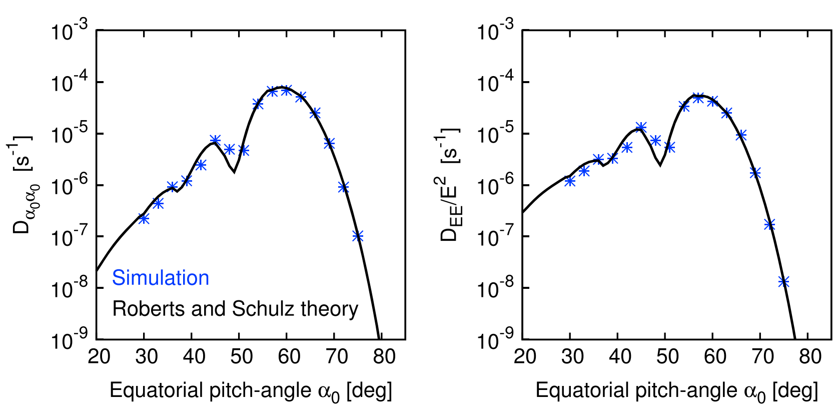

Figure 2. The comparison of Roberts and Schulz diffusion coefficients (left) $D_{\alpha\alpha}$ and (right) D_{EE}/E^2, represented by solid lines, with numerical diffusion coefficients from simulation, represented by blue stars, for E = 1 MeV and f_{pe}/f_{ce} = 3.

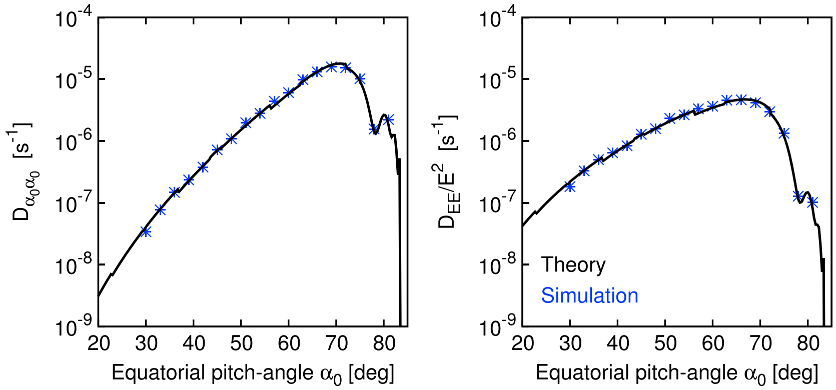

Figure 3. The same as Figure 2, except that the theory is for spatially confined waves.

At $f_{pe}/f_{ce} = 10$, the maximum $D_{\alpha_0\alpha_0}$ and D_{EE}/E^2 from bounce resonance are close to 10^{-3} and 10^{-4}, respectively. These values are much larger than those from gyroresonance [*Horne et al.*, 2007, Figure 5]. We note from Figure 4 that pitch angle bounce resonance diffusion coefficients near $\alpha_0 \sim 90°$ are very small; hence, bounce resonance diffusion is not effective in transporting equatorially mirroring electrons. On the other hand, *Chen et al.* [2015] suggested that nonlinear interactions between a monochromatic magnetosonic wave and electrons through bounce resonance might be effective in transporting 90° electrons. Furthermore, the bounce resonance diffusion by magnetosonic waves is not effective in scattering electrons into the loss cone; therefore, if acting alone, this process should lead mainly to energization of relativistic electrons. On the other hand, similar to gyroresonance, bounce resonance diffusion coefficients of magnetosonic waves peak around some intermediate pitch angles between 50° and 70°. Therefore, a combination of gyroresonance with whistler mode hiss and bounce resonance with magnetosonic waves might significantly reduce the lifetime of electrons in plasmasphere [*Meredith et al.*, 2009; *Mourenas et al.*, 2013]. Finally, based on the comparison of diffusion coefficients, we conclude that for relativistic electrons, the pitch angle and energy diffusion from bounce resonance with magnetosonic waves is as important as that from gyroresonance. Note that although for simplicity we assumed a single wave normal angle in our calculation, adopting a Gaussian distribution in $\tan\psi$ should not change our conclusion qualitatively.

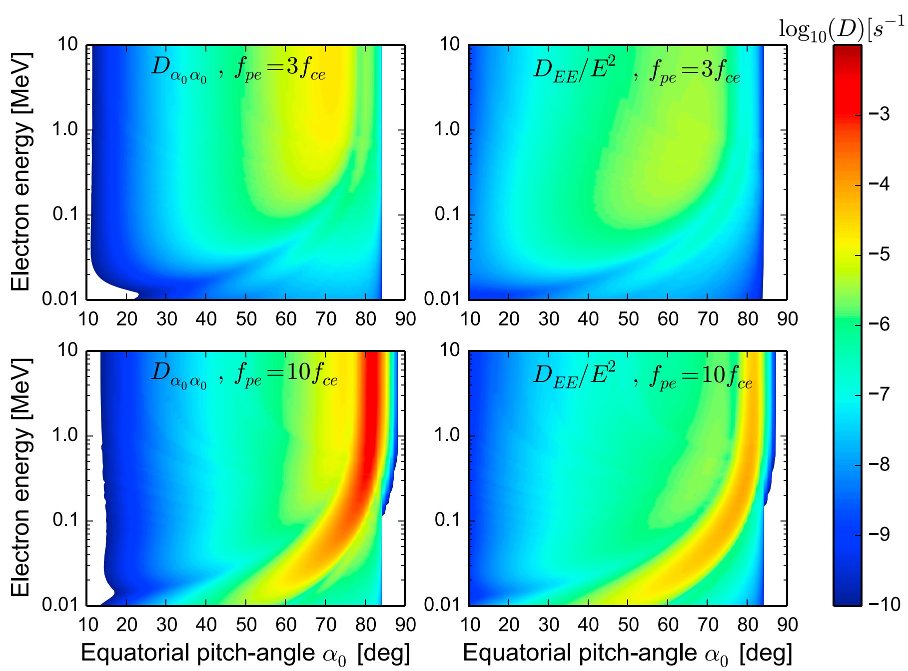

Figure 4. Theoretical bounce resonance diffusion coefficients (left) $D_{\alpha\alpha}$ and (right) D_{EE}/E^2 for (top) $f_{pe} = 3f_{ce}$ and (bottom) $f_{pe} = 10f_{ce}$, for spatially confined magnetosonic waves.

4. Summary and Discussion

In this study, we first reviewed the derivation of the Roberts and Schulz bounce resonance diffusion coefficients using the method of *Schulz and Lanzerotti* [1974]. The Roberts and Schulz diffusion coefficients, however, assume that the wave field covers the whole bounce trajectory of particles, which is not consistent with observations of magnetosonic waves. We then followed the approach by *Schulz and Lanzerotti* [1974] and derived the bounce resonance diffusion coefficients for spatially confined waves. Using guiding center test particle simulations, we validated both sets of theoretical bounce resonance diffusion coefficients for magnetosonic waves. From the calculated pitch angle diffusion coefficients, magnetosonic waves can lead to significant pitch angle scattering of electrons with α_0 near 50°–70°; therefore, when combined with whistler mode hiss waves, bounce resonance might significantly reduce the lifetime of electrons in the plasmasphere [*Meredith et al.*, 2009; *Mourenas et al.*, 2013]. More importantly, we find that the energy diffusion coefficients from bounce resonance with spatially confined magnetosonic waves are comparable to those from gyroresonance. Therefore we conclude that bounce resonance with magnetosonic waves may play an important role in both pitch angle scattering and acceleration of the relativistic electrons in the event studied by *Horne et al.* [2007].

The effect of bounce resonance with magnetosonic waves on electron dynamics, however, depends on various parameters; e.g., plasma density and wave amplitude. We demonstrated in Figure 4 that an increase in plasma density from $f_{pe}/f_{ce} = 3$ to $f_{pe}/f_{ce} = 10$ can increase the maximum bounce resonance diffusion coefficients by about 2 orders of magnitude. Our study should be combined with further observations of magnetosonic wave distribution and plasma parameter information to further estimate the role of bounce resonance with magnetosonic waves in radiation belt electron dynamics.

In this work, we assumed that the magnetosonic waves have a single wave normal angle for a given frequency for simplicity. It is more realistic to allow a spread in wave normal angle as in *Horne et al.* [2007]. Also, we assumed a broadband wave field, which allows us to use diffusion to describe the wave particle interaction process. Observations show that magnetosonic waves might also have fine structures, which might lead to coherent motions of electrons [see, e.g., *Chen et al.*, 2015; *Artemyev et al.*, 2015]. These issues, however, are outside the scope of this study and will be addressed in future investigations.

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