1	Generation of rising-tone chorus in a two-dimensional mirror
2	field by using the general curvilinear PIC code
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14 Abstract

Recently, the generation of rising-tone chorus has been implemented with 15 one-dimensional (1-D) particle-in-cell (PIC) simulations in an inhomogeneous 16 background magnetic field, where both the propagation of waves and motion of 17 18 electrons are simply forced to be parallel to the background magnetic field. In this paper, we have developed a two-dimensional(2-D) general curvilinear PIC simulation 19 code, and successfully reproduced rising-tone chorus waves excited from an 20 anisotropic electron distribution in a 2-D mirror field. Our simulation results show 21 22 that whistler waves are mainly generated around the magnetic equator, and continuously gain growth during their propagation toward higher-latitude regions. The 23 rising-tone chorus waves are formed off the magnetic equator, which propagate 24 25 quasi-parallel to the background magnetic field with the wave normal angle smaller than 25° . Due to the propagating effect, the wave normal angle of chorus waves is 26 increasing during their propagation toward higher-latitude regions along an enough 27 curved field line. The chirping rate of chorus waves are found to be larger along a 28 field line with a smaller curvature. 29

30

31 **1. Introduction**

32	Chorus waves are one of the most significant electromagnetic emissions in the
33	Earth's magnetosphere, which are typically observed within the frequency range of
34	0.1 to $0.8 f_{ce}$ (where f_{ce} is the equatorial electron gyrofrequency) [Burtis and Helliwell,
35	1969; Tsurutani and Smith, 1974, 1977; Meredith et al., 2001; Li et al., 2012]. There
36	is usually a power gap near 0.5 f_{ce} , separating chorus waves into two distinct spectral
37	bands: lower band (0.1-0.5 f_{ce}) and upper band (0.5-0.8 f_{ce}) [Tsurutani and Smith,
38	1974; Koons and Roeder, 1990; Gao et al., 2016a, 2017]. Chorus waves typically
39	exhibit rising or falling tones, but are often accompanied by a hiss-like band in the
40	frequency-time spectrogram [Pope, 1963; Cornilleau-Wehrlin et al., 1978; Koons,
41	1981; Santolík et al., 2009; Li et al., 2012; Gao et al., 2014a]. Besides typical chorus
42	waves, the large amplitude whistler-mode chorus waves are also frequently observed
43	in Van Allen radiation belts during magnetically active periods, which are found to be
44	quasi-monochromatic [Cattell et al., 2008; Cully et al., 2008; Kellogg et al., 2011;
45	Wilson et al., 2011]. Based on statistical results from THEMIS probes, chorus waves
46	in the inner magnetosphere are detected with an amplitude range extending from ~10
47	pT to ~1 nT [Li et al., 2011b, 2012; Gao et al., 2014b]. For lower-band chorus waves,
48	rising tones are mainly field-aligned with the wave normal angle typically smaller
49	than 30°, while falling tones are usually very oblique [Li et al., 2011a, 2012].

50 Chorus waves have been widely believed to play a key role in controlling 51 electron dynamics in the Van Allen radiation belt [Horne et al., 2003; Bortnik and

Thorne, 2007; Summers et al., 2007; Thorne, 2010]. Through efficient pitch angle 52 scattering, chorus waves can remove abundant lower-energy (0.1-30 keV) electrons 53 54 from the magnetosphere, leading to enhanced electron precipitation into the Earth's atmosphere [e.g., Lorentzen et al., 2001; Thorne et al., 2005; Lam et al., 2010]. Both 55 theoretical and observational evidence have shown that chorus waves are the most 56 important candidate for causing Earth's diffuse auroral precipitation [e.g., Thorne et 57 al., 2010; Ni et al., 2011a; Ni et al., 2011b]. Furthermore, acceleration by chorus 58 waves is the primary source of relativistic electrons in the outer radiation belt, which 59 60 can enhance the flux of relativistic electrons (MeV) by several orders within one day during geomagnetic storms [Summers et al., 1998; Meredith et al., 2001; Horne et al., 61 2005; Thorne et al., 2013]. The relativistic electron acceleration can result from 62 63 quasi-linear wave-particle interaction with lower amplitude whistler waves [e.g., Thorne et al. 2013, Li et al. 2014] or from much faster nonlinear electron interaction 64 (trapping and phase bunching) with large amplitude whistler waves [Omura et al. 65 66 2007; Cattell et al., 2008; Agapitov et al., 2014; Artemyev et al., 2014, 2016; Foster et 67 al. 2017].

The generation of chorus waves with frequency chirping (either rising or falling) has long been an attractive topic. Their primary source region is located in the vicinity of the geomagnetic equatorial plane between the plasmapause and magnetopause [LeDocq et al., 1998; Lauben et al., 2002; Santolik et al., 2005; Li et al., 2009]. It has been generally accepted that anisotropic electrons with energy of 10s keV provide the free energy to excite chorus waves in the Earth's magnetosphere [Nunn, 1971;

Karpman et al., 1974; Omura et al., 2008, 2012; Shklyar and Matsumoto, 2009; Li et 74 al., 2010; Demekhov, 2011; Omura and Nunn, 2011; Nunn and Omura, 2012; Gao et 75 76 al., 2014a; Nunn and Omura, 2015]. Many previous works indicated that the frequency chirping of chorus waves should be resulted from nonlinear interactions 77 between resonant electrons and a coherent whistler wave [Nunn, 1974; Nunn et al., 78 1997; Omura and Matsumoto, 1982; Omura and Summers, 2006; Gao et al., 2016b]. 79 Omura et al. [2008] derived the relativistic second-order resonant condition for a 80 field-aligned chorus wave with the frequency chirping, and pointed out that nonlinear 81 82 wave trapping of resonant electrons results in the formation of a resonant current, which causes the growth of rising-tone chorus waves. However, the generation 83 process of chorus waves in a realistic magnetic field configuration with curved field 84 85 lines still remains unclear.

Particle-in-cell (PIC) simulation model is a very promising way to investigate the 86 nonlinear physics underlying the generation process of chorus waves. With a 87 one-dimensional (1-D) PIC simulation model, Hikishima and Omura [2012] studied 88 the excitation of rising-tone chorus waves by anisotropic electrons, and also revealed 89 the production of electron holes resulted from the nonlinear trapping of resonant 90 electrons, which has been predicted by the nonlinear theory [Omura et al., 2008]. 91 Moreover, they confirmed the optimum condition for generating chorus waves, which 92 93 is further supported by THEMIS observations [Gao et al., 2014a]. With the same 94 simulation model, Katoh and Omura [2013] investigated the effect of the background magnetic field inhomogeneity on the generation of rising-tone chorus waves, and 95

showed that the larger spatial inhomogeneity of the background magnetic field results 96 in the larger threshold amplitude for the chorus excitation. In their simulation model, 97 98 in order to include the mirror motion of electrons in the 1-D system, they used a cylindrical mirror field model when solving the motion equation of electrons. 99 Moreover, due to the limitation of 1-D simulation model, both the propagation of 100 chorus waves and motion of electrons are artificially forced to be along the 101 background magnetic field. Whereas chorus waves are typically observed with a finite 102 wave normal angle in the magnetosphere, which indicates both the generation and 103 104 propagation of chorus waves should be studied in a 2-D simulation model.

In this paper, we develop a two-dimensional (2-D) general curvilinear PIC 105 simulation code, and perform it to study the generation of rising-tone chorus in the 106 two-dimensional (2-D) mirror field. In this model, both the propagation of waves and 107 motion of electrons are now allowed to cross the magnetic field. The global 108 distribution of the wave amplitude, frequency chirping rate, wave normal angle, and 109 Poynting flux for the excited rising-tone chorus waves are thoroughly investigated 110 with this code. In Section 2, we will describe our simulation model in detail. The 111 simulation results are presented in Section 3. In Section 4, we summarize and further 112 discuss our principal results. 113

114 **2. Simulation Model**

In the present study, we develop a 2-D general curvilinear PIC simulation code toextend the existing 1-D self-consistent simulation model performed by Katoh and

Omura [2006] and Tao [2014], , and utilize this code to investigate the generation of rising-tone chorus in a 2-D mirror field. The mirror field is considered as the background magnetic field \mathbf{B}_0 in our simulation system, and its three components in Cartesian coordinates are

 $\begin{cases} B_{0x} = -2\xi xz B_{0eq}, \\ B_{0y} = 0, \\ B_{0z} = (1 + \xi z^2) B_{0eq}. \end{cases}$ (1)

where B_{0eq} represents the magnetic field at z=0 (the magnetic equator), and ξ is a parameter representing the inhomogeneity of the background magnetic field. The background magnetic field satisfies $\nabla \cdot \mathbf{B}_0 = 0$, and the magnetic field line equation can be obtained by solving

126
$$\frac{dx}{B_{0x}} = \frac{dz}{B_{0z}},$$
 (2)

127 and it gives

128
$$x = \frac{x_0}{1 + \xi z^2},$$
 (3)

129 where x_0 is the position of the magnetic field lines at z = 0.

In our simulations, the electron population consists of two components: cold and hot components. The cold electrons are treated as a fluid, and hot electrons are modelled as kinetic particles considering the relativistic effect, which is the same as those used in the electron hybrid model of Katoh and Omura [2007]. Ions are assumed to be an immobile neutralizing component since the frequencies of the investigated chorus waves are much larger than the ion cyclotron frequency. The motions of cold electrons are governed by the following equations:

137
$$\frac{\partial n_c}{\partial t} = -\nabla \cdot \left(n_c \mathbf{V}_c \right),$$

138

$$\frac{\partial \mathbf{V}_c}{\partial t} = -(\mathbf{V}_c \cdot \nabla) \mathbf{V}_c + \frac{q_e}{m_e} (\mathbf{E} + \mathbf{V}_c \times \mathbf{B}),$$

$$\mathbf{J}_{c} = q_{e} n_{c} \mathbf{V}_{c}$$

,

where the subscript "c" refers to the cold electron component, n is the electron
number density, V is the electron bulk velocity, J is the electron current density,
B and E are the total magnetic and electric fields, respectively. The hot electrons
move in the Lorentz force:

(4)

144
$$\frac{d(\gamma \mathbf{v}_{h})}{dt} = \frac{q_{e}}{m_{e}} (\mathbf{E} + \mathbf{v}_{h} \times \mathbf{B}), \qquad (5)$$

145 where the subscript "h" refers to the hot electron component, \mathbf{v}_h is the particle 146 velocity of hot electrons, and $\gamma = 1/\sqrt{1 - (|\mathbf{v}_h|/c)^2}$. The total current density can be 147 calculated by

148
$$\mathbf{J} = \mathbf{J}_c + q_e n_h \mathbf{V}_h, \tag{6}$$

149 where n_h is the hot electron number density, and V_h is the average velocity of hot 150 electrons. Then, the magnetic and electric fields are updated by solving Maxwell 151 equations

$$\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\mu_0 \varepsilon_0} \nabla \times \mathbf{B} - \frac{1}{\varepsilon_0} \mathbf{J},$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}.$$
(7)

152

In our simulation, we use the algorithm to deposit the charge density and currentdensity to the grids, which automatically satisfies the charge conservation.

155 Our simulation is performed in the x-z plane. The simulation domain are divided

into grids by a group of field lines $x = x_0/(1+\xi z^2)$ with equal spacing Δx_0 at the 156 magnetic equator and a group of lines $z = z_0$ with equal spacing Δz_0 as shown in 157 Figure 1. The spatial scale in the simulation is normalized by $\rho_{e0} = w_{\parallel eq} / \Omega_{e0}$, where 158 $w_{\parallel eq}$ and Ω_{e0} are the initial values of parallel thermal velocity of hot electrons and 159 electron gyrofrequency at the magnetic equator, respectively. The domain boundaries 160 are $x = \pm 700 \rho_{e0} / (1 + \xi z^2)$ and $z = \pm 500 \rho_{e0}$. We move the particles in Cartesian 161 coordinate, however, the wave magnetic and electric fields are updated by solving 162 Maxwell equations in general curvilinear coordinate (p, w): 163

164
$$\begin{cases} p = (1 + \xi z^2) x\\ w = z. \end{cases}$$
 (8)

165 The scale factors are

166
$$h_p = \frac{1}{1 + \xi w^2}$$
, and $h_w = \sqrt{\left(2\xi pw/\left(1 + \xi w^2\right)^2\right)^2 + 1}$. (9)

Two types of boundary conditions are used in our simulation. Reflecting 167 boundary conditions are used for particles. When a particle 168 hits the $x = \pm 700 \rho_{e0} / (1 + \xi z^2)$ or $z = \pm 500 \rho_{e0}$ boundaries, its normal velocity 169 (perpendicular to the boundary) reverses the direction. Absorbing boundary conditions 170 are assumed for waves. A masking function is used to damp wave fields near the 171 simulation boundary [Tao, 2014]. 172

Initially, the cold electrons is assumed to be distributed uniformly in the simulation domain. Their number density is n_{c0} , and bulk velocity is $V_{c0} = 0$. The plasma frequency of cold electrons is $\omega_{pe} = 5\Omega_{e0}$, where $\omega_{pe} = \sqrt{n_{c0}e^2/m_e\varepsilon_0}$. For the hot electrons, at the magnetic equator (z = 0), the number density is n_{heq} , and 177 velocity distribution is bi-Maxwellian,

178
$$f_{eq}(v_{h\parallel}, v_{h\perp}) = \frac{1}{(2\pi)^{3/2}} \exp\left(-\frac{v_{h\parallel}^2}{2w_{\parallel eq}^2} - \frac{v_{h\perp}^2}{2w_{\perp eq}^2}\right),$$
(10)

where $w_{\Box eq}$ and $w_{\perp eq}$ are the thermal velocities of hot electrons in the parallel and perpendicular directions, respectively. According to the force balance condition [Chan et al., 1994], the hot electrons pressure are be expressed as:

182
$$p_{\parallel} = \frac{n_{heq}T_{\parallel eq}}{\zeta}$$
, and $p_{\perp} = \frac{n_{heq}T_{\perp eq}}{\zeta^2}$, (11)

where $\zeta = 1 + (T_{\perp eq}/T_{\parallel eq} - 1)(1 - B_{0eq}/B_0)$, $T_{\parallel eq}$ and $T_{\perp eq}$ are the parallel and perpendicular temperatures of hot electrons at the magnetic equator. Based on conservation of energy and magnetic moment, the hot electrons number density derived from direct integration of Eq. (10) has the following dependence

187
$$n_h = n_{heq} / \zeta , \qquad (12)$$

188 Then the hot electrons temperature off the equator are

189
$$T_{\parallel} = T_{\parallel eq}$$
, and $T_{\perp} = T_{\perp eq} / \zeta$, (13)

190 Therefore, the velocity distribution of hot electrons can be expressed as

191
$$f(v_{h\parallel}, v_{h\perp}) = \frac{1}{(2\pi)^{3/2}} \frac{1}{w_{\parallel eq}} \frac{1}{w_{\perp eq}^2/\zeta} \exp\left(-\frac{v_{h\parallel}^2}{2w_{\parallel eq}^2}\right) \exp\left(-\frac{v_{h\perp}^2}{2w_{\perp eq}^2/\zeta}\right).$$
(14)

The initial settings of hot electrons at the magnetic equator are as follows: the number density is $n_{heq} = 0.01n_{c0}$, the temperature anisotropy is $T_{\perp eq}/T_{\parallel eq} = 4.5$, and the parallel beta $\beta_{\parallel} = n_{heq}T_{\parallel eq}/(B_{0eq}^2/2\mu_0) = 0.02$. The initial number density and temperature anisotropy of hot electrons are shown in Figure 2. Approximate 1.05 billion superparticles are used for hot electrons in our simulation. The parameters are summarized in Table 1.

198 **3. Simulation Results**

199 The overview of our simulation results is presented in Figure 3, which shows spatial profiles of $\delta(T_{\perp}/T_{\parallel})$ and amplitude of fluctuating magnetic fields 200 perpendicular to the background magnetic field B_{\perp}/B_{0eq} at $\Omega_{e0}t = 400, 700$ and 1200, 201 respectively. Here, $\delta(T_{\perp}/T_{\parallel}) = (T_{\perp}/T_{\parallel})_t - (T_{\perp}/T_{\parallel})_0$ is the variation of temperature 202 anisotropy of hot electrons compared to the initial value. At $\Omega_{e0}t = 400$, the 203 temperature anisotropy of hot electrons, which is considered as the energy source of 204 chorus waves, begins to decrease at the magnetic equator, as shown in Figure 3a. Just 205 as expected, the fluctuating magnetic field also begins to grow from the magnetic 206 equator in Figure 3b. With the time increasing, from $\Omega_{e0}t = 400$ to 1200, the decrease 207 of the temperature anisotropy becomes more significant at the magnetic equator, and 208 209 also gradually expands to higher-latitude regions. Meanwhile, the fluctuating magnetic fields are firstly excited at the magnetic equator, and then propagate up to 210 the higher-latitude regions. 211

In order to further analyze the detailed frequency-time spectrogram of chorus waves and study effects from the inhomogeneity and curved configuration of the background magnetic field, we extract time series of fluctuating magnetic fields from ten locations along two typical magnetic field lines L1 and L2, which have been marked by symbols "*" in Figure 3f. Figure 4 displays the frequency-time spectrogram of fluctuating magnetic field perpendicular to the background magnetic field at these ten locations (|z|=0, 80, 160, 240 and $320 \rho_{e0}$ along L1 and L2). At the

magnetic equator (Figures 4i and 4j), there are only whistler mode waves, rather than 219 chorus waves with the coherent structure. However, off the magnetic equator (Figures 220 221 4a-f), there are clear rising-tone chorus elements excited in our simulation. Even though these two filed lines are close to each other, we can still find one distinct 222 difference between them: two chorus elements are observed along L2 (right column), 223 but only one chorus elements can be found along L1 (left column). Figure 5 (top) 224 presents one rising-tone element observed at location $z = -160 \rho_{e0}$ along field line 225 L1 whose wave packet is exhibited in Figure 5 (bottom). The quasi-periodic 226 227 modulation of the wave amplitude and the resulting subpackets can be observed, which are similar to the observations and 1-D simulation results [Santolík et al., 2003; 228 229 Tao et al., 2017].

230 The evolution of fluctuating magnetic fields perpendicular to the background magnetic field along two filed lines (L1 and L2) is illustrated in Figures 6. The short 231 bars in Figures 6a and 6b denote the start and end of rising-tone elements at locations 232 233 |z| = 160 and 240 ρ_{e0} along two field lines. The main source of these chorus waves is from the magnetic equator region. Afterwards, wave packets will propagate toward 234 the polar region and their amplitudes are gradually enhanced. More interestingly, the 235 wave packets confined within two short bars at |z| = 160 and $240 \rho_{e0}$ seem to come 236 from the same source region, i.e., the magnetic equator, which means the rising-tone 237 element is first formed at the magnetic equator but only with a negligible amplitude. 238 239 Then, the rising-tone element will propagate away from the equator, and its amplitude will continuously increase up to the observable level. 240

The distribution of chirping rates of rising-tone elements is displayed in Figure 7. 241 Here, along both field lines (L1 and L2), we choose two typical locations (|z| = 160242 and 240 ρ_{e0}) to estimate the chirping rate. In each panel, the black dotted line along 243 the rising-tone element is a fitting of dominant frequencies with the maximum power 244 at each time by the linear least square method. Then the chirping rate is just the slope 245 of the black dotted line. The chirping rates Γ are estimated as 3.7×10^{-4} , 3.5×10^{-4} 246 Ω_{e0}^2 at z = -160, $-240 \rho_{e0}$ on L1, and as 7.7×10^{-4} , $7.6 \times 10^{-4} \Omega_{e0}^2$ at z = 160, 247 240 ρ_{e0} on L2, respectively. With the same distance off the magnetic equator, the 248 249 chirping rate of the rising-tone element along L1 are smaller than that along L2. Figure 8 presents the distribution of the chirping rates Γ of rising-tone elements at 250 $|z| = 160 \rho_{e0}$ in different field lines. The $|x_0|$ denotes the distance from these field 251 252 lines to the middle field line at the equator. There is a trend that the chirping rate decreases as $|x_0|$ increases. 253

With the same format of Figure 4, Figure 9 shows the distribution of wave 254 normal angles of rising-tone chorus along field lines L1 and L2 at ten locations. For 255 visual clarity, we only display wave normal angles θ of wave modes whose wave 256 power is larger than $0.25 P_{\text{max}}$, where the P_{max} is the maximum of wave power at a 257 fixed position during the simulation time. Furthermore, the average wave normal 258 angles of rising-tone elements at different places along field lines (left) L1 and (right) 259 L2 are displayed in Figures 10. The rising-tone elements along field line L2 are nearly 260 field-aligned with the wave normal angles $\theta \approx 8^{\circ}$ as seen in right panels of Figures 9 261 and 10. While the wave normal angles of chorus waves increase as the waves 262

propagate toward higher-latitude regions along the curved field line L1 as shown in left panels of Figures 9 and 10. And the wave normal angles can increase up to ~25° when the chorus waves reach the absorption region at $|z| > 360 \rho_{e0}$.

We also consider the distribution of the Poynting flux in this paper. Figure 8 266 illustrates the parallel component of Poynting vector $\mathbf{S}_{\parallel}/|\mathbf{S}|$ of chorus emissions at 267 ten selected locations along two field lines L1 and L2, where |S| is the intensity of 268 Poynting flux **S** for each wave mode. For visual clarity, we only display $S_{\parallel} / |S|$ of 269 wave modes whose wave power is larger than $0.04P_{\text{max}}$. At the magnetic equator, z =270 0, the waves are propagating toward both upward and downward (Figures 8i-j), which 271 is consistent with the prediction of the linear theory [Gary and Karimabadi, 2006]. At 272 higher-latitude region, |z| = 80 and $160 \rho_{e0}$, the waves are mainly propagating toward 273 274 the polar region, but we can still find that some wave modes are propagating toward the magnetic equator (Figures 8e-h). This indicates the anisotropic electron 275 distribution off the equator is also unstable to excite whistler waves, which is 276 consistent with the expanded region of the declining anisotropy in Figure 3e. At the 277 higher-latitude regions, the overwhelming majority of the waves propagate toward the 278 polar region. It is worth noting that the rising-tone chorus remains the same 279 propagating direction after it is excited at the magnetic equator. 280

4. Summary and Discussion

In order to consider the effects of both the inhomogeneity and curved configuration of the background magnetic field on the evolution of whistler waves,

we develop a 2-D general curvilinear PIC simulation code, where both the 284 propagation of waves and motion of electrons are now allowed to cross the magnetic 285 286 field. We successfully reproduced rising-tone chorus waves excited by an anisotropic electron distribution in a 2-D mirror field. Our simulation results show that whistler 287 waves are mainly generated around the magnetic equator, and continuously gain 288 growth during their propagation toward higher-latitude regions. The rising-tone 289 chorus waves are formed off the magnetic equator, which propagate quasi-parallel to 290 the background magnetic field with the wave normal angle smaller than 25° . There is 291 a clear trend that the wave normal angles of chorus waves are increasing during their 292 propagation toward higher-latitude regions along an enough curved field line. And the 293 chirping rate of chorus waves are found to be larger along a field line more close to 294 295 the middle field line.

In the theoretical work by Helliwell [1967], the change of Γ mainly depends on 296 the intensity B_0 and inhomogeneity dB_0/ds of the background field in the source 297 region of chorus waves, where s is a distance along the field line from the equatorial 298 plane. However, the variations of both B_0 and dB_0/ds due to the different $|x_0|$ in 299 these source regions in our simulation are small and unable to account for the change 300 of Γ over $|x_0|$. There is a clear trend that the magnetic amplitude of chorus waves 301 will decrease with the distance $|x_0|$ from the middle filed line at the magnetic equator 302 (not shown). Therefore, we speculated that the larger chirping rate on L2 than L1 may 303 304 be due to the higher wave amplitude at the magnetic equator, which can be supported by the theoretical work by Omura et al. [2008]. 305

Compared with previous 1-D PIC simulation results, our new simulation model 306 exhibits several interesting properties of chorus waves in a 2-D frame. Due to the 307 propagating effect, the wave normal angles of chorus waves are increasing during 308 their propagation toward the higher-latitude regions along a enough curved field line 309 as shown in Figures 9 and 10. This is qualitatively consistent with the result obtained 310 from the ray tracing model [Bortnik et al., 2011; Breuillard et al., 2012; Chen et al., 311 2012] and observation results [Li et al., 2011b; Agapitov et al., 2013]. In previous 1-D 312 PIC simulation works, only the magnetic field line L2 is considered in their models 313 314 [Omura et al., 2008; Hikishima and Omura, 2012; Kato and Omura, 2013; Tao, 2014]. However, along a curved magnetic field line (L1), the chirping rate of chorus waves is 315 observed to be smaller than that along L2 (Figure 7) in our 2-D simulation model. 316 317 Moreover, the threshold of electron temperature anisotropy to generate rising-tone chorus is significantly reduced after the assumptions of parallel propagation of chorus 318 waves and parallel motion of electrons are released in a 2-D PIC simulation. With 319 almost same plasma parameters, the electron temperature anisotropy $T_{\perp eq}/T_{\parallel eq}$ to 320 excite rising-tone chorus in a 1-D PIC simulation is required to be ~7 [Tao, 2014], 321 322 while that is reduced to ~3.8 in our 2-D PIC simulation. This may suggest that the nonlinear interaction of chorus waves with hot electrons in the 2-D simulation is more 323 efficient than that in the 1-D simulation. At this stage, our PIC code is only developed 324 in a 2-D mirror field, which can be considered as an extension of previous simulation 325 works. For the more practical purpose, we are trying to implement our PIC code in a 326 2-D or even 3-D dipole field. 327

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571 Figure captions:

- Figure 1. The sketch map of spatial grids of the simulation domain in Cartesian
 coordinates (x, z). The blue lines denote the magnetic field lines.
- Figure 2. Initial distribution of (a) the hot electrons number density n_h/n_{c0} , and (b) the anisotropy of hot electrons T_{\perp}/T_{\parallel} in the simulation domain, respectively.

Figure 3. The spatial profiles of the disturbed anisotropy $\delta(T_{\perp}/T_{\parallel})$ and perpendicular fluctuating magnetic fields B_{\perp}/B_{0eq} at $\Omega_{e0}t = 400$, 700 and 1200, respectively. In Figure 3f, the curve and straight solid lines through $(-186\rho_{e0}, 0)$ and (0, 0) represent two selected typical magnetic field lines marked by L1 and L2, respectively. Ten observation locations at |z| = 0, 80, 160, 240 and 320 ρ_{e0} along the field line L1 (L2) are denoted by symbols "*", respectively.

582 Figure 4. The frequency-time spectrogram of perpendicular fluctuating magnetic field

583 B_{\perp}/B_{0eq} at ten selected locations along L1 and L2 as shown in Figure 3f.

Figure 5. (top) Frequency-time spectrogram of rising-tone element at location $z = -160\rho_{e0}$ along field line L1. (bottom) Wave packet from $\Omega_{e0}t = 300$ to 1100 corresponding to the duration between the two vertical dashed lines plotted in the top panel.

Figure 6. Temporal evolution of perpendicular fluctuating magnetic fields B_{\perp}/B_{0eq} along field lines (a) L1 and (b) L2, respectively. The short bars in two panels denote the start and end of rising-tone elements at locations |z| = 160 and 240 ρ_{e0} along two field lines.

Figure 7. The frequency-time spectrogram of perpendicular fluctuating magnetic field B_{\perp}/B_{0eq} at |z| = 160 and 240 ρ_{e0} along L1 and L2, respectively. In each panel, the black dotted line along the rising-tone element is a fitting of dominant frequencies with the maximum power at each time by the linear least square method.

Figure 8. The distribution of the chirping rates Γ of rising-tone elements appearing at $|z| = 160 \rho_{e0}$ for different field lines. The $|x_0|$ denotes the distance from these field lines to the middle field line at the equator.

Figure 9. The distribution of the wave normal angle θ at ten selected locations along field lines L1 and L2, respectively.

601 Figure 10. The average wave normal angle θ of rising-tone elements at different

- locations along field lines (left) L1 and (right) L2.
- **Figure 11**. The distribution of the parallel component of Poynting vector $\mathbf{S}_{\parallel}/|\mathbf{S}|$ at
- ten selected locations along two field lines L1 and L2, respectively.

Figure 1.



Figure 2.



Figure 3.



Figure 4.



Figure 5.



Figure 6.



Figure 7.



Figure 8.



Figure 9.



Figure 10.



Figure 11.

