

SANG Longlong, WU Mingyu, LU Quanming. Electrostatic structure of the electron phase-space holes generated by the electron two-stream instability with a finite width. *Chin. J. Space Sci.*, 2017, **37**(5): 517-523. DOI:10.11728/cjss2017.05.517

# Electrostatic Structure of the Electron Phase-space Holes Generated by the Electron Two-stream Instability with a Finite Width\*

SANG Longlong WU Mingyu LU Quanming

(CAS Key Laboratory of Geospace Environment, School of Earth and Space Sciences, University of Science and Technology of China, Hefei 230026)

**Abstract** Space satellite observations in an electron phase-space hole (electron hole) have shown that bipolar structures are discovered at the parallel cut of parallel electric field, while unipolar structures spring from the parallel cut of perpendicular electric field. Particle-in-cell (PIC) simulations have demonstrated that the electron bi-stream instability induces several electron holes during its nonlinear evolution. However, how the unipolar structure of the parallel cut of the perpendicular electric field formed in these electron holes is still an unsolved problem, especially in a strongly magnetized plasma ( $\Omega_e > \omega_{pe}$ , where  $\Omega_e$  is defined as electron gyrofrequency and  $\omega_{pe}$  is defined as plasma frequency, respectively). In this paper, with two-dimensional (2D) electrostatic PIC simulations, the evolution of the electron two-stream instability with a finite width in strongly magnetized plasma is investigated. Initially, those conditions lead to monochromatic electrostatic waves, and these waves coalesce with each other during their nonlinear evolution. At last, a solitary electrostatic structure is formed. In such an electron hole, a bipolar structure is formed in the parallel cut of parallel electric field, while a unipolar structure presents in the parallel cut of perpendicular electric field.

**Key words** Electron phase-space hole, Two-stream instability, PIC simulation

**Classified index** P 353

## 0 Introduction

Since Geotail satellite, which focused on the Earth's magnetotail, observed Electrostatic Solitary Waves (ESWs) in 1994<sup>[1]</sup>, plenty of observing evidences flow out for various space environments, such as the fore-shock region of the bow shock<sup>[2]</sup>, the auroral region<sup>[3]</sup>, the solar wind<sup>[4]</sup>, the magnetosheath<sup>[5]</sup>, and the separatrix region of magnetic reconnection<sup>[6-8]</sup>. These observed ESWs have common features that besides the parallel cut of parallel electric field has a bipolar structure, its perpendicular component has a unipo-

lar structure<sup>[9,10]</sup>.

ESWs are modeled as a format of electron phase-space hole by Bernstein *et al.*<sup>[11]</sup>. These structures can be calculated from the Vlasov and Poisson equations as the stationary solutions of the Bernstein-Greene-Kruskal (BGK) model<sup>[12,13]</sup>. Particle-in-cell (PIC) simulations in one-dimension (1D) have confirmed that the electron bi-stream instability would induce ESWs of several electron holes in the simulation domain during its nonlinear evolution, and the parallel cut of electric field in the parallel component at these ESWs has a bipolar structure<sup>[14-22]</sup>. Also,

\* Supported by the National Science Foundation of China(41474125, 41331067, 41421063), 973 Program (2013CBA01503), and Key Research Program of Frontier Sciences, CAS (QYZDJ-SSW-DQC010)

Received October 16, 2016. Revised March 07, 2017

E-mail: sandar@mail.ustc.edu.cn

the electron velocity distributions in these ESWs consist of trapped and passed particles, and there is a hole in the phase space, which is consistent with the results of Bernstein *et al.*<sup>[14,18,23,24]</sup>. Recently, with two-dimensional (2D) PIC simulations, ESWs are found to swing due to the effect of transverse instability<sup>[25–27]</sup>. The transverse instability is demonstrated to be a kind of self-focusing instability: perturbations in ESWs lead to the electric potentials' transverse gradients, which focuses the trapped electrons into the areas with the superfluous electrons, and then the transverse instability occurs<sup>[28]</sup>. In a weakly magnetized plasma ( $\Omega_e < \omega_{pe}$ , where  $\Omega_e$  is defined as electron gyrofrequency and  $\omega_{pe}$  is defined as plasma frequency), the transverse instability will break a 1D electron hole into several 2D electron holes. In these 2D electron holes, the parallel cut of the electric fields in the parallel component and perpendicular cut of the electric field in the perpendicular components have bipolar and unipolar structures, respectively<sup>[25]</sup>.

In a strongly magnetized plasma, the parallel cut of the parallel electric field has a bipolar structure in an electrons hole, however, whether a unipolar structure of the perpendicular electric field can be formed in an electron hole during the electron two-stream instability is still unknown<sup>[25]</sup>. In this paper, the nonlinear evolution of the electron two-stream instability with a finite width is investigated with the help of 2D electrostatic PIC simulations. It is found that besides the bipolar structure of parallel electric field, the unipolar structure of electric field in the perpendicular component can also be formed in an electron hole.

## 1 Simulation Model

In this paper, 2D electrostatic PIC simulations with periodic boundary conditions in  $x$  and  $y$  directions are performed to investigate the nonlinear evolution of electron bi-stream instability in a strongly magnetized plasma<sup>[29]</sup>. In the electrostatic simulation mo-

del, we set a uniform background magnetic field along the  $x$  direction. Our simulations also employ two different electron components with an initial drift velocity between them, and the Maxwellian distribution is satisfied for these two different electron components, which have the same thermal velocity  $v_{te}$  ( $v_{te} = \sqrt{k_B T_e / m_e}$ , where  $T_e$  is the electron temperature). The drift velocity  $V_d$  between the two components is  $4.0 v_{te}$ , *i.e.*  $V_d = 4.0 v_{te}$ , which is along the  $x$  direction. The time, distance and velocity are normalized by  $\omega_{pe}^{-1}$  ( $\omega_{pe} = \sqrt{n_0 e^2 / m_e \varepsilon_0}$ , and  $n_0$  is the total electron number density), Debye length  $\lambda_D$  ( $\lambda_D = (\varepsilon_0 T_e / n_0 e^2)^{-1/2}$ ), and  $v_{te}$ , respectively. The electric field is normalized by  $m_e \omega_{pe} v_{te} / e$ . The cell size is  $\lambda_D \times \lambda_D$ , and the time step  $\Delta t = 0.02 \omega_{pe}^{-1}$ . The simulation uses a scale of grids of  $512 \times 256$ . The mass ratio of ion to electron  $m_i/m_e = 1836$ , and  $\Omega_e/\omega_{pe} = 2$  and 10.

## 2 Simulation Results

In this paper, four cases are performed: in Case A and C, the electron beam initially flows in the parallel direction with an infinite width; while in Case B and D the initial electron beam has a finite width of  $32 \lambda_D$ , and it only exists in the region of  $112\lambda_D < y < 144\lambda_D$ . In Case A and B,  $\Omega_e/\omega_{pe} = 2$ , and in Case C and D,  $\Omega_e/\omega_{pe} = 10$ .

Figure 1 presents the time evolutions of the electric field components  $E_x$  and  $E_y$  at  $\omega_{pe}t = 400, 1000$ , and 1924 for Case A. Nearly monochromatic waves are firstly excited. Then, these waves coalesce with each other. At  $\omega_{pe}t = 1000$ , from the distribution of  $E_x$ , it can be found that there are only two electrostatic solitary structures in the simulation area. At the same time, in these solitary structures,  $E_y$  forms a streaked structure. At  $\omega_{pe}t = 1924$ , there is only one electrostatic solitary structure for  $E_x$  in the simulation domain, and the streaked structure of  $E_y$  occupies the whole simulation domain. Such a process of the formation of the solitary electrostatic structure for  $E_x$  and excitation of streaked structure for  $E_y$

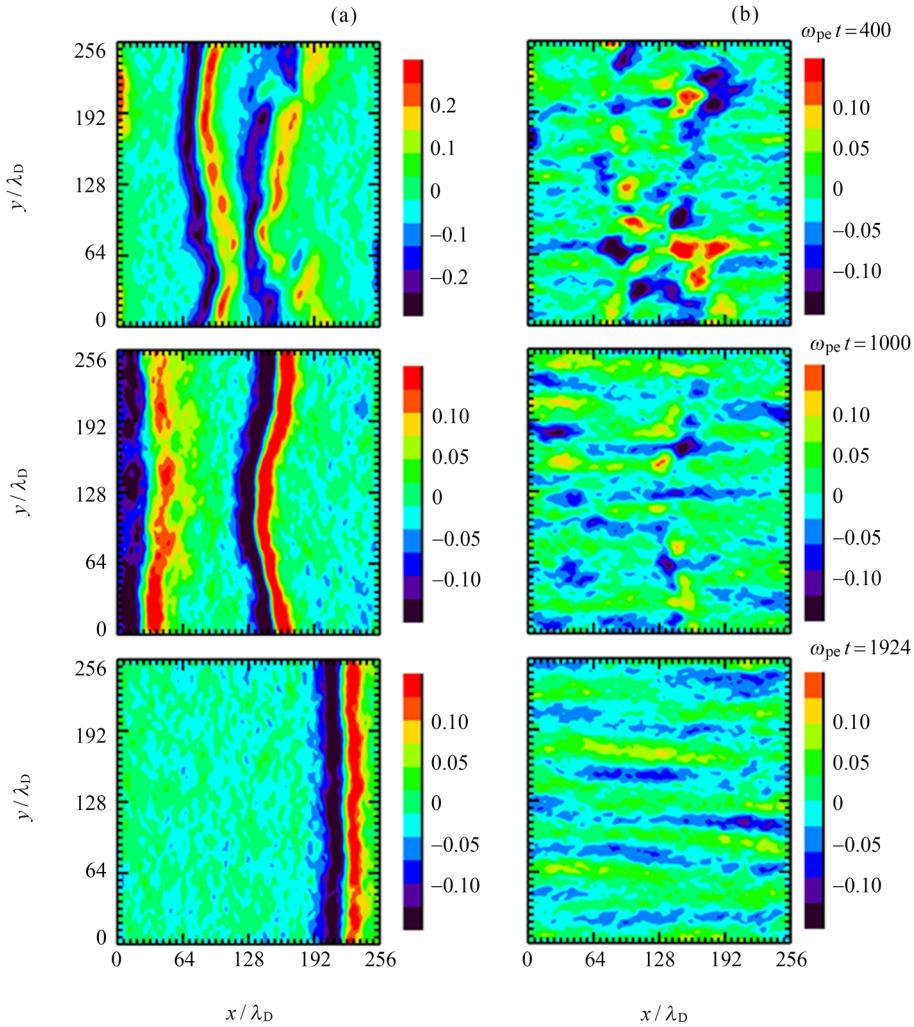


Fig. 1 Time history of the electric field components  $E_x$  (a) and  $E_y$  (b) at  $\omega_{pe}t = 400, 1000, 1924$  for Case A

have been described previously in detail by Lu *et al.*<sup>[25]</sup>, which is considered to be associated with electrostatic whistler waves emitted in the solitary structure. Obviously, the parallel cut of the electric field in the parallel component in the electron hole has a bipolar structure, however, the unipolar structure for the parallel cut of the electric field in the perpendicular component cannot be observed.

Figure 2 plots the time evolution of the electric field components  $E_x$  and  $E_y$  at  $\omega_{pe}t = 400, 1000, 1788$  for Case B. Initially, from  $E_x$ , we can find that nearly monochromatic waves are excited in the region where the electron beam exists. Then, as in Case A, these waves also coalesce with each other. At  $\omega_{pe}t = 1000$ , from  $E_x$ , we can find that there is only one electro-

static solitary structure, where  $E_y$  has a negative value in the lower part but a positive value in the upper part. Such structures of  $E_x$  and  $E_y$  can be seen more clearly at  $\omega_{pe}t = 1788$ , and they could last for more than one thousand electron plasma periods. Figure 3 plots the parallel cut of  $E_x$  and  $E_y$  along  $y = 136\lambda_D$  and  $y = 120\lambda_D$  at  $\omega_{pe}t = 1788$  for Case B. Obviously, in the solitary structure, a bipolar structure is formed in the parallel cut of the electric field in the parallel component, and a unipolar structure presents in the parallel cut of the perpendicular electric field.  $E_y$  has a positive value in the solitary structure along about  $y = 136\lambda_D$ , and it has a negative value along about  $y = 120\lambda_D$ . This is easy to be understood, because an electron hole has a positive potential. When the

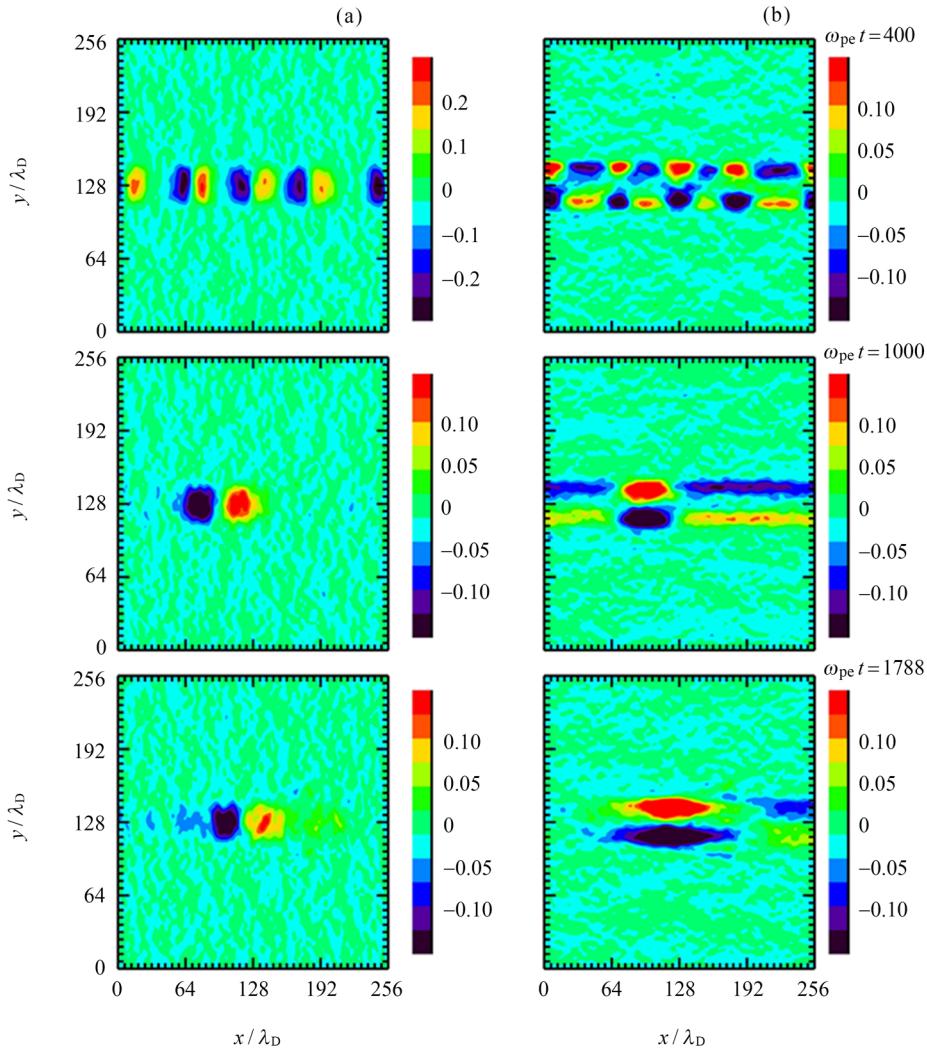


Fig. 2 Time history of the electric field components  $E_x$  (a) and  $E_y$  (b) at  $\omega_{pe}t = 400, 1000, 1788$  for Case B

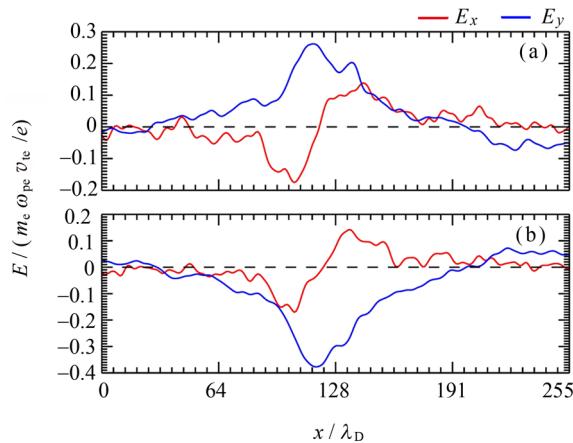


Fig. 3 Profiles of  $E_x$  and  $E_y$  along  $y = 136\lambda_D$  (a) and  $y = 120\lambda_D$  (b) at  $\omega_{pe}t = 1788$  for Case B

electron beam has a finite width, the positive potential also has a finite width along the  $y$  direction in the electron hole. Therefore,  $E_y$  has a negative value in the lower part but positive value in the upper part of the electron hole.

Figure 4 and 5 show the time evolution of electric field  $E_x$  and  $E_y$  at  $\omega_{pe}t = 400, 1000, 2000$  for Case C and D, respectively. Similar to Case A and B, from  $E_x$ , we can find that one solitary structure is formed at last. However, only when the electron beam has a finite width, the parallel cut of the electric field  $E_y$  has a unipolar structure in the solitary structure. Therefore, we can conclude that in strongly magnetized plasma during the nonlinear evolution

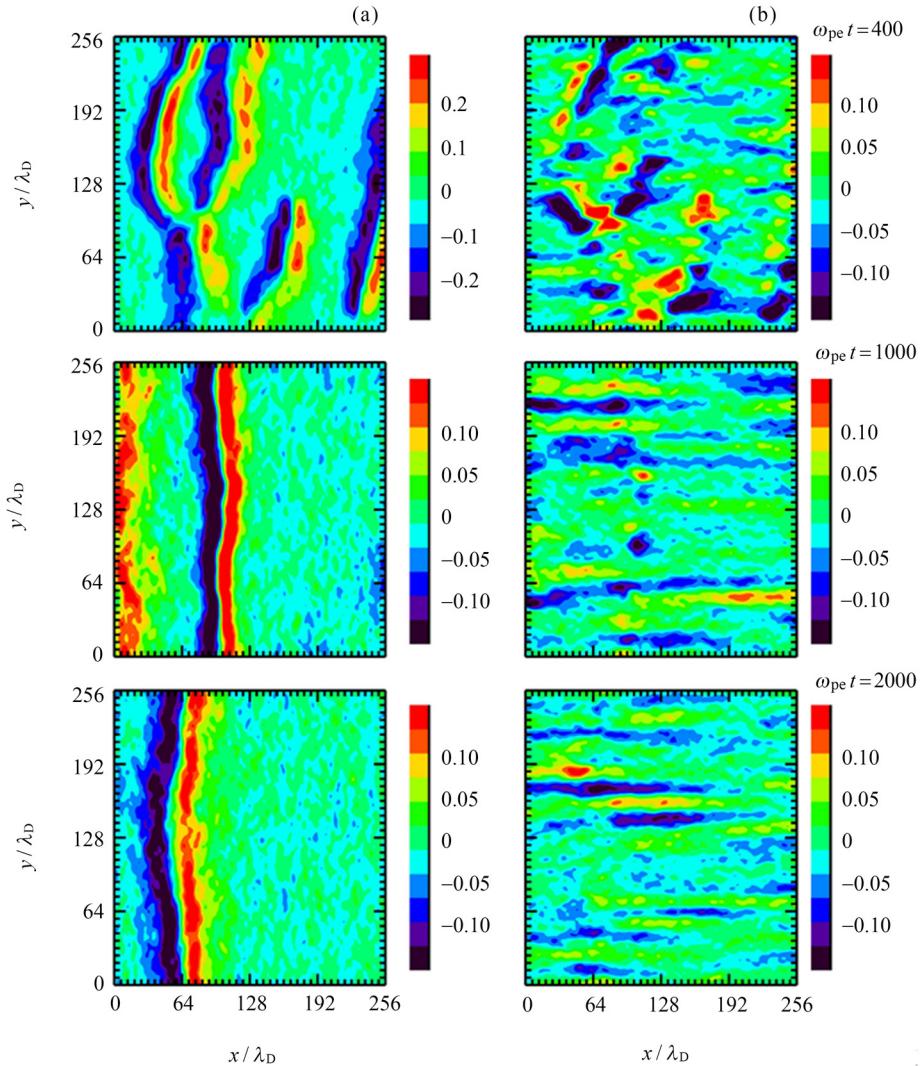


Fig. 4 Time history of the electric field components  $E_x$  (a) and  $E_y$  (b) at  $\omega_{pe} t = 400, 1000, 2000$  for Case C

stage of the electron bi-stream instability with a finite width, a solitary electrostatic structure can be formed, where the parallel cuts of the electric fields in the parallel and perpendicular directions have a bipolar and unipolar structures, respectively.

### 3 Conclusions and Discussion

In this paper, with a 2D electrostatic PIC simulation model, the electron two-stream instability's nonlinear evolution in strongly magnetized plasma is investigated, and the results of the electron beam existing in the whole simulation domain are compared with that of the electron beam with a finite width. Our results

show that the evolution of the parallel electric field is similar, and the solitary electric structure is formed at the late stage of the simulation through the coalescence of the excited waves, where the parallel cut of the electric field in parallel component has a bipolar structure. However, the evolution of the electric field in the perpendicular component is different. In cases where the electron beam exists in the whole simulation domain, the electric field in the perpendicular component forms a streaked structure. In cases where the electron beam has a finite width, the parallel cut of the electric field in the perpendicular component has a unipolar structure in the solitary electrostatic structure.

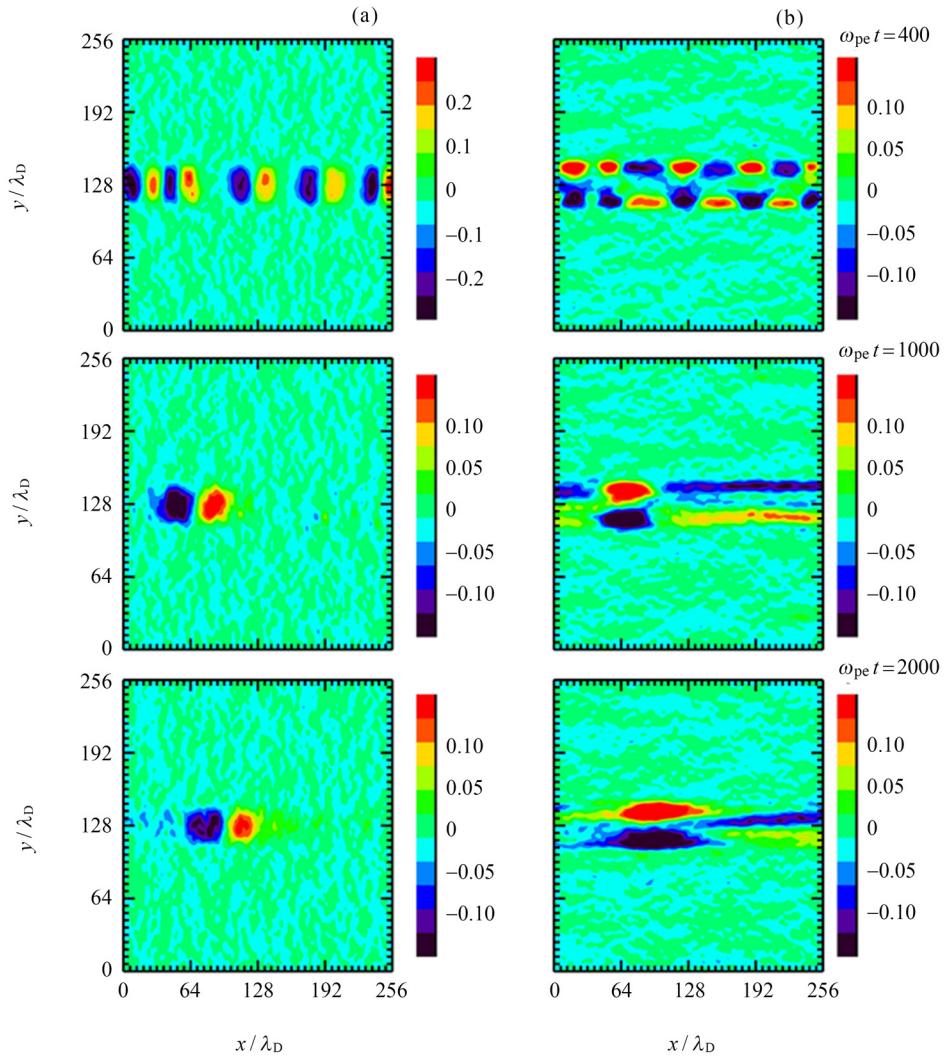


Fig. 5 Time history of the electric field components  $E_x$  (a) and  $E_y$  (b) at  $\omega_{pe}t = 400, 1000, 2000$  for Case D

Satellite observations have shown that ESWs usually have a bipolar structure of the electric field in the parallel component and a unipolar structure of the electric field in the perpendicular component<sup>[9,10]</sup>. Our simulation results have demonstrated that the formed ESWs during their nonlinear evolution may have a bipolar structure of the electric field in the parallel component and a unipolar structure of the electric field in the perpendicular component when the electron two-stream instability has a finite width. The electron beams generated in geophysical environment, for example in magnetic reconnection, usually have a finite width<sup>[30–32]</sup>. Therefore, our results may explain the observed structures of both the parallel and perpendicular component of the electric fields in

geophysical environment.

## References

- [1] MATSUMOTO H, KOJIMA H, MIYATAKE T, et al. Electrostatic Solitary Waves (ESW) in the magnetotail: BEN wave forms observed by GEOTAIL [J]. *Geophys. Res. Lett.*, 1994, **21**(25): 2915-2918
- [2] BALE S D, KELLOGG P J, LARSEN D E, et al. Bipolar electrostatic structures in the shock transition region: evidence of electron phase space holes [J]. *Geophys. Res. Lett.*, 1998, **25**(15): 2929-2932
- [3] ERGUN R E, CARLSON C W, MCFADDEN J P, et al. FAST satellite observations of large-amplitude solitary structures [J]. *Geophys. Res. Lett.*, 1998, **25**(12): 2041-2044
- [4] MANGENEY A, SALEM C, LACOMBE C, et al. WIND observations of coherent electrostatic waves in the solar wind [J]. *Ann. Geophys.*, 1999, **17**(3): 307-320

- [5] PICKETT J S, CHEN L J, KAHLER S W, *et al.* Isolated electrostatic structures observed throughout the Cluster orbit: relationship to magnetic field strength [J]. *Ann. Geophys.*, 2004, **22**(7): 2515-2523
- [6] WANG Rongsheng, LU Quanming, KHOTYAITSEV Y V, *et al.* Observation of double layer in the separatrix region during magnetic reconnection [J]. *Geophys. Res. Lett.*, 2014, **41**(14): 4851-4858
- [7] LI S Y, OMURA Y, LEMBÈGE B, *et al.* Geotail observation of counter directed ESWs associated with the separatrix of magnetic reconnection in the near-Earth magnetotail [J]. *J. Geophys. Res.*, 2014, **119**(1): 202-210
- [8] GRAHAM D B, KHOTYAITSEV Y V, VAIVADS A, *et al.* Electrostatic solitary waves with distinct speeds associated with asymmetric reconnection [J]. *Geophys. Res. Lett.*, 2015, **42**(2): 215-224
- [9] ERGUN R E, CARLSON C W, MCFADDEN J P, *et al.* Debye-scale plasma structures associated with magnetic-field-aligned electric fields [J]. *Phys. Rev. Lett.*, 1998, **81**(4): 826-829
- [10] FRANZ J R, KINTNER P M, PICKET J S, *et al.* Polar observations of coherent electric field structures [J]. *Geophys. Res. Lett.*, 1998, **25**(8): 1277-1280
- [11] BERNSTEIN I B, GREENE J M, KRUSKAL M D. Exact nonlinear plasma oscillations [J]. *Phys. Rev.*, 1957, **108**(3): 546-550
- [12] CHEN L J, PICKETT J, KINTNER P, *et al.* On the width-amplitude inequality of electron phase space holes [J]. *J. Geophys. Res.*, 2005, **110**(A9): A09211
- [13] NG C S, BHATTACHARJEE A, SKIFF F. Weakly collisional Landau damping and three-dimensional Bernstein-Green-Kruskal modes: new results on old problems [J]. *Phys. Plasmas*, 2006, **13**(5): 055903
- [14] OMURA Y, KOJIMA H, MATSUMOTO H. Computer simulation of electrostatic solitary waves: a nonlinear model of broadband electrostatic noise [J]. *Geophys. Res. Lett.*, 1994, **21**(25): 2923-2926
- [15] GOLDMAN M V, OPPENHEIM M M, NEWMAN D L. Nonlinear two-stream instabilities as an explanation for auroral bipolar wave structures [J]. *Geophys. Res. Lett.*, 1999, **26**(13): 1821-1824
- [16] OPPENHEIM M, NEWMAN D L, GOLDMAN M V. Evolution of electron phase-space holes in a 2D magnetized plasma [J]. *Phys. Rev. Lett.*, 1999, **83**(12): 2344-2347
- [17] LU Quanming, WANG Shui, DOU Xiankang. Electrostatic waves in an electron-beam plasma system [J]. *Phys. Plasmas*, 2005, **12**(7): 072903
- [18] LU Q M, WANG D Y, WANG S. Generation mechanism of electrostatic solitary structures in the Earth's auroral region [J]. *J. Geophys. Res.*, 2005, **110**(A3): A03223
- [19] UMEDA T, OMURA Y, MIYAKE T, *et al.* Nonlinear evolution of the electron two-stream instability: two-dimensional particle simulations [J]. *J. Geophys. Res.*, 2006, **111**(A10): A10206
- [20] WU Mingyu, LU Quanming, ZHU Jie, *et al.* The magnetic structures of electron phase-space holes formed in the electron two-stream instability [J]. *Astrophys. Space Sci.*, 2012, **338**(1): 81-85
- [21] WU Mingyu, LU Quanming, ZHU Jie, *et al.* Electromagnetic particle-in-cell simulations of electron holes formed during the electron two-stream instability [J]. *Plasma Sci. Technol.*, 2013, **15**(1): 17-24
- [22] JAO C S, HAU L N. Fluid aspects of electron streaming instability in electron-ion plasmas [J]. *Phys. Plasmas*, 2014, **21**(2): 022103
- [23] DU Aimin, WU Mingyu, LU Quanming, *et al.* Transverse instability and magnetic structures associated with electron phase space holes [J]. *Phys. Plasmas*, 2011, **18**(3): 032104
- [24] WU Mingyu, LU Quanming, DU Aimin, *et al.* The evolution of the magnetic structures in electron phase-space holes: two-dimensional particle-in-cell simulations [J]. *J. Geophys. Res.*, 2011, **116**(A10): A10208
- [25] LU Q M, LEMBEGE B, TAO J B, *et al.* Perpendicular electric field in two-dimensional electron phase-holes: a parameter study [J]. *J. Geophys. Res.*, 2008, **113**(A10): A11219
- [26] WU Mingyu, LU Quanming, HUANG Can, *et al.* Transverse instability and perpendicular electric field in two-dimensional electron phase-space holes [J]. *J. Geophys. Res.*, 2010, **115**(A10): A10245
- [27] WU Mingyu, WU Hong, LU Quanming, *et al.* Effects of perpendicular thermal velocities on the transverse instability in electron phase space holes [J]. *Chin. Phys. Lett.*, 2010, **27**(9): 095201
- [28] MUSCHIETTI L, ROTH I, CARLSON C W, *et al.* Transverse instability of magnetized electron holes [J]. *Phys. Rev. Lett.*, 2000, **85**(1): 94-97
- [29] LU Quanming, CAI Dongsheng. Implementation of parallel plasma particle-in-cell codes on PC cluster [J]. *Comput. Phys. Commun.*, 2001, **135**(1): 93-104
- [30] FU X R, LU Q M, WANG S. The process of electron acceleration during collisionless magnetic reconnection [J]. *Phys. Plasmas*, 2006, **13**(1): 012309
- [31] LU Quanming, HUANG Can, XIE Jinlin, *et al.* Features of separatrix regions in magnetic reconnection: comparison of 2-D particle-in-cell simulations and Cluster observations [J]. *J. Geophys. Res.*, 2010, **115**(A11): A11208
- [32] HUANG Can, LU Quanming, WANG Peiran, *et al.* Characteristics of electron holes generated in the separatrix region during antiparallel magnetic reconnection [J]. *J. Geophys. Res.*, 2014, **119**(8): 6445-6454