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Key Points:

- We develop a 1D PIC-δf simulation model embedding a continuous injection of energetic electrons in a dipole magnetic field
- It is demonstrated that a continuous injection of energetic electrons is essential for the repetitive emissions of chorus waves
- An intense injection of energetic electrons will lead to a decrease of the time separation between the chorus elements

Supporting Information:

Supporting Information may be found in the online version of this article.

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Repetitive Emissions of Rising-Tone Chorus Waves in the Inner Magnetosphere

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Abstract Chorus waves in the inner magnetospheres of the Earth and other magnetized planets typically occur in the form of discrete and repetitive quasi-monochromatic emissions with a frequency chirping, which was discovered more than 50 years ago. However, until now there is still no satisfactory explanations for repetitive emissions of chorus waves. In this Letter, chorus emissions excited by energetic electrons with a temperature anisotropy are studied by a one-dimensional PIC-δf simulation in a dipole magnetic field, and it is demonstrated that a continuous injection of energetic electrons caused by an azimuthal drift is essential for the repetitive emissions of chorus waves. Consistent with satellite observations, both discrete and continuous spectra can be reproduced. An intense injection of energetic electrons will lead to a decrease of the time separation between the chorus elements, and the chorus emissions evolve from a discrete to a continuous spectrum when the injection is sufficiently strong.

Plain Language Summary Chorus waves are electromagnetic emissions that are commonly observed in the Earth's inner magnetosphere, and they typically occur in the form of discrete and repetitive quasi-monochromatic emissions with a rising frequency chirping. The rising frequency chirping of chorus waves are considered to be related with electromagnetic electron holes in the wave phase space formed after resonant electrons have been trapped by whistler-mode waves. However, until now there is still no satisfactory explanations for repetitive emissions of chorus waves. We develop a one-dimensional PIC-δf simulation model to study the excitation of chorus waves by energetic electrons with a temperature anisotropy in a dipole magnetic field, and a continuous injection of energetic electrons is embedded in the model. The repetitive emissions of chorus waves with a rising frequency chirping are generated in the simulations, and it is demonstrated clearly that the continuous injection of energetic electrons caused by an azimuthal drift is the reason for the repetitive emissions of chorus waves. It is further found that an intense injection of energetic electrons will lead to a decrease of the time separation between the chorus elements, and a continuous spectrum is formed when the injection is sufficiently strong.

1. Introduction

Chorus waves are electromagnetic emissions that are commonly observed in geospace consisting of ionized gas (plasma) embedded with a magnetic field, and known for an ensemble of distinct elements with each showing frequency chirping (Burtis & Helliwell, 1969; Li et al., 2012; Meredith et al., 2001; Tsurutani & Smith, 1974, 1977). Such frequency chirping in every element is often featured with a rising frequency (over ~1 kHz) over a short duration of period (~0.1 s). Chorus waves are so named to reflect similar frequency-time variation to birds' dawn chorus in acoustic mode. Such electromagnetic chorus waves have well-known for their significant roles in geospace phenomena, to name a few, acceleration of electrons to relativistic energy and thus the formation of Earth's radiation belts (Horne et al., 2005; Summers et al., 1998; Thorne et al., 2013; Xiao et al., 2009); precipitation of electrons into the ionosphere, and thus generation of diffuse and pulsating aurora (Lam et al., 2010; Lorentzen et al., 2001; Ni et al., 2008; Thorne et al., 2005, 2010). The exhibition of a series of short-living microburst often seen in electron precipitation is generally considered to be caused by the repetitive emissions of chorus elements (Hikishima et al., 2010; Mozer et al., 2018; Tsurutani et al., 2013).

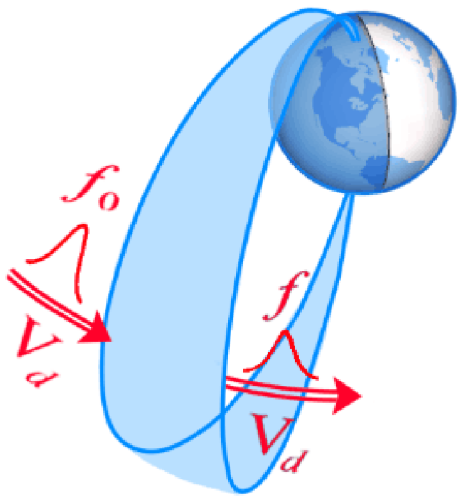

Figure 1. The schematic diagram for the evolution of energetic electrons in a dipole magnetic field. Due to an azimuthal drift, energetic electrons with a distribution f_0 is injected into the simulation domain, while particles leaving the domain have a distribution f. The blue-shaded region is the region of chorus wave excitation.

Rising-tone chorus waves have been successfully reproduced in particle-in-cell (PIC) and Vlasov simulations, and their formation is considered to be fulfilled through nonlinear wave-particle interaction (Hikishima & Omura, 2012; Katoh & Omura, 2006, 2007; Ke et al., 2017; Lu et al., 2019; Nunn et al., 1997; Omura & Nunn, 2011; Omura et al., 2009; Tao, 2014). An electromagnetic electron hole in the wave phase space is formed after resonant electrons have been trapped by whistler-mode waves, and then a resonant current is generated when the background magnetic field has a spatial inhomogeneity, which at last results in rising-tone chorus waves (Hikishima & Omura, 2012; Omura & Summers, 2006; Omura et al., 2009; Omura & Nunn, 2011). However, these PIC simulations were performed in an isolated system, where electrons are forced to be bounded along a single field line. Such an isolated system is not realistic because in reality the electrons are subject to azimuthal drift across the field line where chorus elements are excited. For these simulations, only one or several distinct elements of rising-tone chorus have been observed since the free energy stored in the electron distribution is released in a bursty way, and the repetitive feature as typically shown in satellite observations cannot be reproduced (Ke et al., 2017). In this Letter, with a one-dimensional (1-D) PIC-δf simulation, in which a continuous injection of energetic electrons with a temperature anisotropy caused by the azimuthal drift is implemented, we demonstrate the generation mechanism of repetitive emissions of rising-tone chorus waves.

2. Simulation Setup

A one-dimensional (1-D) PIC-δf simulation model is developed to study the emissions of rising-tone chorus waves in a dipole magnetic field. In the simulations, there are two electron components: cold and energetic electrons. We split the electron distribution into a background and perturbed parts $f = f_0 + \delta f$, and obtain the evolution of the perturbed electron distribution δf (Sydora, 2003; Tao et al., 2017). In our model, as schematically shown in Figure 1, a continuous injection of energetic electrons caused by an azimuthal drift in a dipole magnetic field is considered. Due to the azimuthal drift, energetic electrons with a distribution f_0 are injected into the simulation domain, and energetic electrons leaving the simulation domain have a distribution f. The time scale related to the injection of energetic electrons can be estimated as the ratio of transverse scale of individual chorus element and electron drift speed.

Initially, the distribution of energetic electrons is assumed to satisfy the bi-Maxwellian function at the equator as follows: the ratio of the number density of energetic electrons to that of cold electrons is $n_{h0}/n_{c0} = 0.6\%$, the ratio of cold electron plasma frequency to electron gyrofrequency is $\omega_{pe}/\Omega_{e0} = 4.97$ (where $\omega_{pe} = \sqrt{n_0 e^2 / m_e \varepsilon_0}$ is the electron plasma frequency, $n_0 = n_{c0} + n_{h0}$, and $\Omega_{e0} = eB_{0\mathrm{eq}}/m_e$ is the electron gyrofrequency at the equator), the temperature anisotropy of energetic electrons is $T_{\perp 0}/T_{\parallel 0} = 6$, and the parallel plasma beta of energetic electrons is $\beta_{\parallel h0} = n_{h0}T_{\parallel 0}/(B_{0\mathrm{eq}}^2/2\mu_0) = 0.01$. These parameters are typical values at $L = 6$, and the other values of these parameters off the equator along the magnetic field can be obtained with the Liouville's theorem (Lu et al., 2019). In the simulations, there are 4,000 grid number, and grid cell is $0.34d_e$ (where $d_e = c/\omega_{pe}$ is the electron inertial length). The topology of the magnetic field in the simulations is roughly equal to that at $L = 0.6$, and the latitude ranges from about $-32°$–$32°$. On average, there are about 4,000 particles per cell, and the time step is $0.03\Omega_{e0}^{-1}$. Absorbing boundary condition is applied for the electromagnetic field, while reflecting boundary is applied for particles. Different from the previous PIC simulations performed in an isolated system (Ke et al., 2017), energetic electrons with a temperature anisotropy are injected continuously into the simulation domain due to the azimuthal drift, which is determined by a time scale τ_D. The details of the model and simulation setup can be found in the supplementary information.

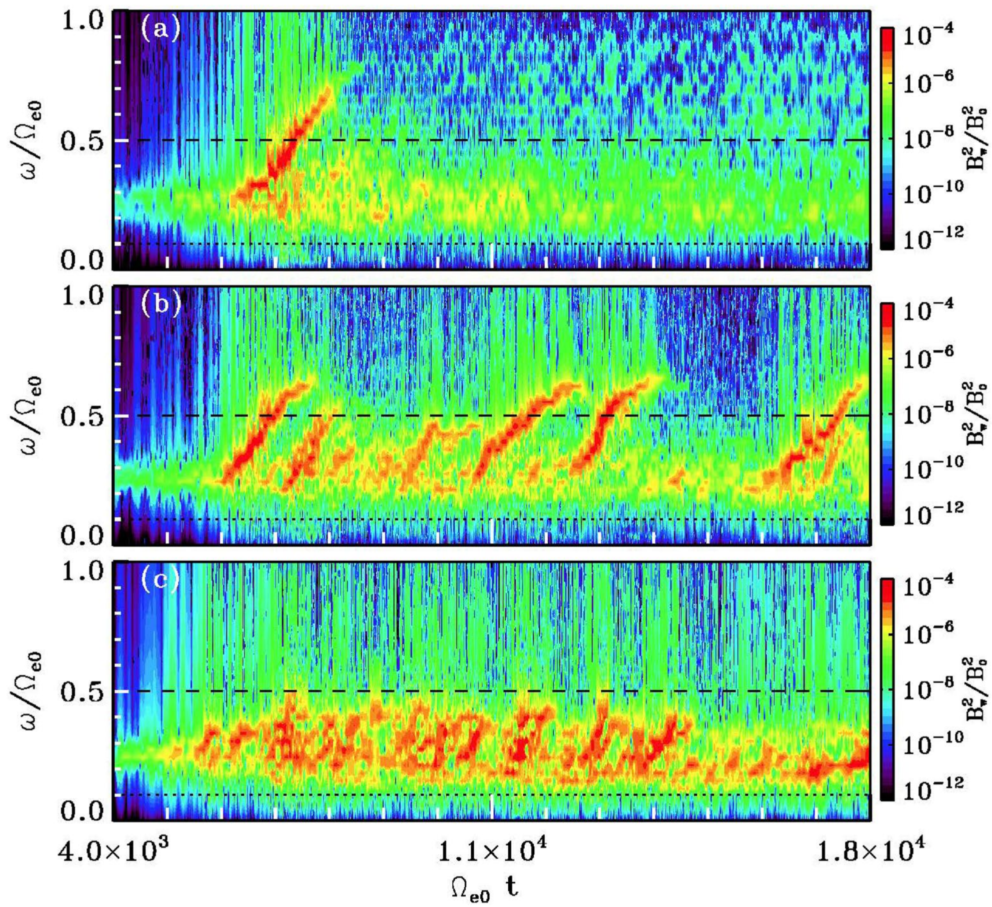

Figure 2. The spectrogram of magnetic power for chorus waves obtained at the latitude $\lambda = 15°$ from the PIC-δf simulations. (a) $\Omega_{e0}\tau_D = \infty$, (b) $\Omega_{e0}\tau_D = 2000$, and (c) $\Omega_{e0}\tau_D = 400$. Here, the dotted and dashed lines in black or white represent $0.1\Omega_{e0}$ and $0.5\Omega_{e0}$, respectively, and Ω_{e0} is the electron gyrofrequency at the equator. The frequencies of these waves range from $\sim 0.2\Omega_{e0}$ to $\sim 0.75\Omega_{e0}$.

3. Simulation Results

As described in previous PIC simulations without a continuous injection of energetic electrons, whistler mode waves are excited by the electron temperature anisotropy around the magnetic equator, and they have frequency chirping with a rising tone after leaving the source region with a limited altitude about 3° (Lu et al., 2019). Figure 2 plots the spectrogram of wave magnetic power at the latitude $\lambda = 15°$, which is obtained from the 1-D PIC-δf simulations. At this latitude, the rising-tone chorus waves are clearly shown. For the case of $\Omega_{e0}\tau_D = \infty$, where energetic electron injection into the system is turned off, only one rising-tone element forms (Figure 2a). This case is similar to that of an isolated system, which has been studied by many previous PIC simulations (Hikishima & Omura, 2012; Ke et al., 2017; Lu et al., 2019; Tao, 2014). In such a situation, the free energy is provided by the initial anisotropic distribution of energetic electrons, and in general only one element of chorus waves is generated. When energetic electrons are injected continuously into the system, the chorus waves exhibit repetitive elements with a rising frequency. With the intensification of the energetic electron injection (through the decrease of $\Omega_{e0}\tau_D$), the time separation between the elements become smaller and smaller. At $\Omega_{e0}\tau_D = 2000$, the chorus waves display an ensemble of discrete and repetitive elements as shown in Figure 2b. The average time separation between the elements is about $1{,}000\Omega_{e0}^{-1}$. If we use the magnetic field at $L = 6$, the time separation is estimated to be around 0.05s. When the injection of energetic electrons is sufficiently intense, the time separation between the elements is even smaller and spectrogram looks like an almost continuous spectrum (Figure 2c). The frequencies of

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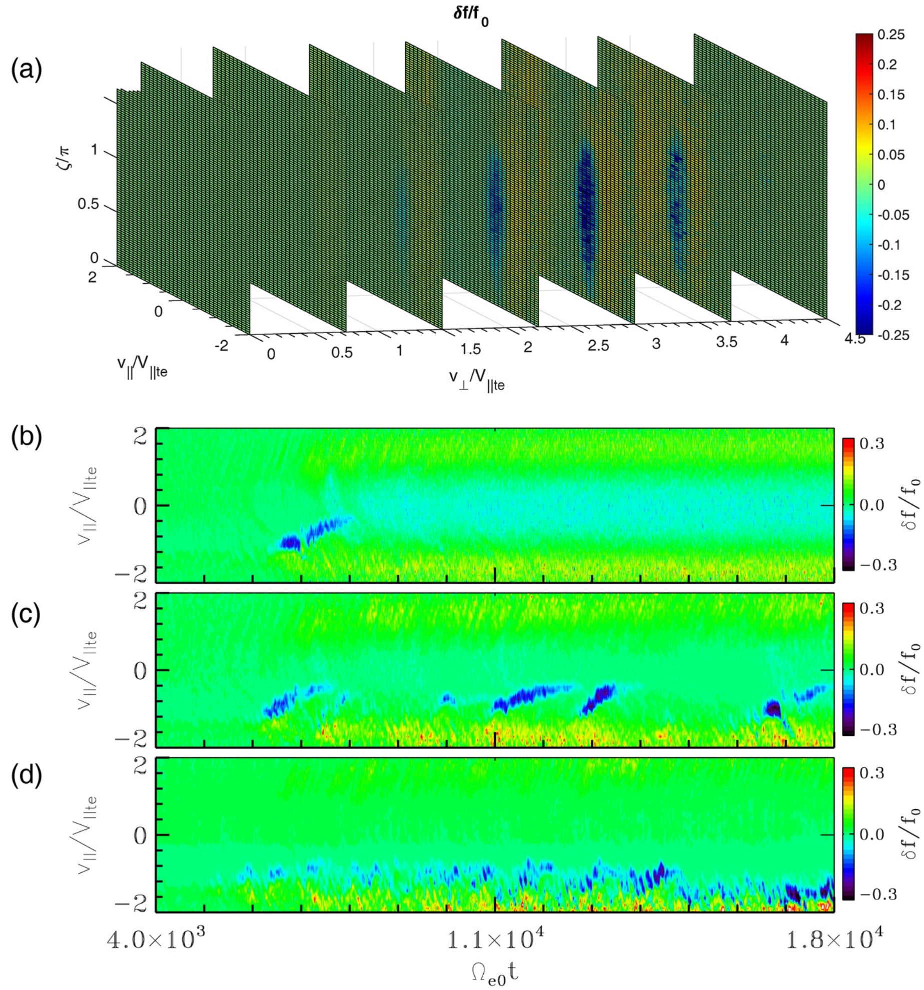

Figure 3. Electron holes associated with chorus waves obtained at the latitude $\lambda = 15^{\circ}$ from the PIC-δf simulations. (a) Electron distributions $\delta f\left(v_{\parallel}, v_{\perp}, \zeta\right)$ at different values of $v_{\perp} \ / \ V_{\parallel te}$ for the selected hole at $\Omega_{e0}t = 6700$ in the case with $\Omega_{e0}\tau_{D} = 2000$. (b–d) The time evolution of electron distributions $\delta f\left(v_{\parallel}\right)$ at $\zeta = 0.8\pi$ and $v_{\perp} \ / \ V_{\parallel te} = 2.25$ in the three cases, $\Omega_{e0}\tau_{D} = \infty$, 2000, and 400, respectively.

these waves range from $\sim 0.2\Omega_{e0}$ to $\sim 0.75\Omega_{e0}$, and the amplitudes of the chorus waves are $B_{w} \ / \ B_{0} \sim 2 \times 10^{-3}$. Assuming $L = 6$, we can know that the amplitude is about 300 pT.

It is generally accepted that chorus waves are excited as a result of electron holes in the wave-particle interaction angle (ζ), which are formed by electrons resonantly trapped in the waves. Similarly, we can also find such kind of electron holes in our PIC-δf simulations. A typical electron hole at $\Omega_{e0}t = 6700$ for the case with $\Omega_{e0}\tau_{D} = 2000$ is shown in Figure 3a by plotting the electron distributions $\delta f\left(v_{\parallel}, \zeta\right)$ at different values of$v_{\perp} \ / \ V_{\parallel te}$ (where ζ is the angle made by electron perpendicular velocity and wave perpendicular magnetic field vector, and $V_{\parallel te}$ is the thermal velocity of energetic electrons). The hole only exists in the range $v_{\perp} \ / \ V_{\parallel te}$ from ~ 0.5 to ~ 6.5 (Figure 3a). By comparing Figures 2a and 3b–3d, we can find that each chorus element is accompanied by an electron hole and their duration in time are comparable.

In our 1-D PIC-δf simulations that target the repetitive nature of chorus waves, the wave emissions are propagating along the magnetic field. A wave power gap at half electron gyrofrequency, which is often observed in the magnetosphere, is not reproduced, and the lack of the gap in the 1D simulation implies that such a gap may be caused by Landau damping as a consequence of oblique propagation.

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4. Discussion

Whistler-mode waves are at first excited by the electron temperature anisotropy in a localized region around the equator, a rising tone emission of chorus waves is then generated when the amplitude of the whistler mode waves satisfies a threshold condition. As pointed out in Omura et al. (2009), the amplitude of chorus waves is determined by the nonlinear growth of the chorus waves and wave attenuation caused by the convection effect due to the inhomogeneity along the magnetic field line, and the nonlinear growth of the chorus waves is related to the resonant electron distribution.

If there is no refilling of energetic electrons to replenish the initial electron distribution, with the frequency increase of the chorus emissions, less and less electrons can resonantly interact with the chorus waves, and the amplitude of the chorus waves becomes smaller and smaller until the cease of the chorus elements. In such a situation, because the available free energy provided by the electron temperature anisotropy is limited, there is only chorus element as described in the case with $\Omega_{e0}\tau_D = \infty$. When anisotropic electrons are injected continuously into the system, another cycle of chorus element formation will start after the free energy provided by the refilled electron distribution become sufficiently large, and this process can repeat again and again. At last, the repetitive emissions of chorus waves are generated.

With the intensification of the electron injection (decreasing τ_D), the time delay to provide the sufficient free energy to generate the next cycle of chorus element become small, therefore, the time separation between the chorus elements decrease until a continuous spectrum is formed.

In the previous PIC and Vlasov simulations of chorus waves in a dipole field, the electrons leaving from the boundaries are assumed to be reflected into the simulation system (Katoh & Omura, 2007; Nunn et al., 1997). In these simulations, several chorus elements may be generated because the supplied electron fluxes from the boundaries still have free energy to excited whistler-mode waves. However, the free energy will be exhausted after several times of the reflection in the boundaries, and then the generation of chorus waves stops. In our simulations, the refilling electron fluxes are caused by an azimuthal drift, and their distribution is kept as the initial one. Therefore, the repetitive emissions of chorus waves are generated because there is always free energy to excite whistler-mode waves.

5. Conclusions

In this Letter, we developed a 1-D PIC-δf simulation model to study chorus emissions in a dipole magnetic field, where the injection of anisotropic energetic electrons is considered. The injection of energetic electrons through the azimuthal drift into the field line of chorus excitation is found to play a critical role to generate the repetitive emissions of rising-tone chorus waves. When the injection is considered, the chorus waves exhibit an ensemble of discrete and repetitive elements, and each element is accompanied by an electromagnetic electron hole in the wave-particle interaction phase space. The time separation between the elements shortens with the intensification of the injection, which is consistent with satellite observations (Gao et al., 2014).

Discrete and repetitive emissions of chorus waves are ubiquitously observed in the Earth's radiation belt, and our PIC-δf simulations show that such kind of repetitive emissions is mediated by the injection of energetic electrons caused by the azimuthal drift in the dipole field. Chorus waves are also observed as a hiss-like emission with an apparently continuous spectrum (Gao et al., 2014). Such continuous spectrum can also be accounted for by the effect of electron injection. When the injection is sufficiently strong, the time separation between elements becomes small in comparison with individual element duration. As a result, the elements will overlap each other and chorus waves of a continuous spectrum form.

Data Availability Statement

Simulation datasets for this research are available at the following link https://doi.org/10.6084/m9.figshare.14480823.

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