GEOPHYSICAL RESEARCH LETTERS, VOL. 30, NO. 20, 2039, doi:10.1029/2003GL017901, 2003

An empirical formula relating the geomagnetic storm's intensity to the interplanetary parameters: $-\overline{VB_z}$ and Δt

Yuming Wang, C. L. Shen, S. Wang, and P. Z. Ye

School of Earth and Space Sciences, University of Science and Technology of China, Hefei, Anhui, China

Received 5 June 2003; revised 7 September 2003; accepted 17 September 2003; published 22 October 2003.

[1] We statistically study 105 geomagnetic storms with a *Dst* peak value ≤ -50 nT during 1998–2001 to examine the influence of the interplanetary parameters $-\overline{VB_z}$ and its duration Δt on the intensity of geomagnetic storms. About 33% of the events are associated with intense storms with $Dst_{min} \leq -100$ nT. It is found that $-\overline{VB_z}$ is much more important than Δt for the formation of geomagnetic storms. A stronger $-\overline{VB_z}$ can produce a more intense storm, whereas a longer Δt can not. A simple empirical formula relating the *Dst* peak value to $-\overline{VB_z}$ and Δt is obtained, which shows a good correlation ($CC = 0.9528$) between the estimate value and the observations. This formula suggests that a compressed B_s field tends to have a more prominent geoeffectiveness. Moreover, we also identify 33 large $-\overline{VB_z}$ intervals with $-\overline{VB_z} > 5$ mV/m and $\Delta t > 3$ hours in the same study interval, and find that they all caused intense storms ($Dst_{min} \leq -100$ nT) and 8/9 of the great storms ($Dst_{min} \leq -200$ nT) were due to interplanetary compressed structures. *INDEX TERMS:* 2134 Interplanetary Physics: Interplanetary magnetic fields; 2784 Magnetospheric Physics: Solar wind/magnetosphere interactions; 2788 Magnetospheric Physics: Storms and substorms. **Citation:** Wang, Y., C. L. Shen, S. Wang, and P. Z. Ye, An empirical formula relating the geomagnetic storm's intensity to the interplanetary parameters: $-\overline{VB_z}$ and Δt, *Geophys. Res. Lett.*, *30*(20), 2039, doi:10.1029/2003GL017901, 2003.

1. Introduction

[2] A geomagnetic storm is due to the enhancement of ring current at the Earth's equator. *Dst* is one of the important indices in evaluating the level of geomagnetic disturbances. The interplanetary causes of geomagnetic storms have been extensively studied [e.g., *Tsurutani et al.*, 1997 and therein]. The Earth-directed solar wind speed (V) and southward component (B_s) of interplanetary magnetic fields are of most importance in creating geomagnetic storms [*Snyder et al.*, 1963; *Fairfield and Cahill*, 1966]. Certainly, a sufficiently long duration of B_s is also necessary.

[3] Based on statistical studies, the interplanetary criteria in creating geomagnetic storms has been concluded. For intense storms with $Dst_{min} \leq -100$ nT, the value of B_s should be greater than 10 nT with its duration $\Delta t \geq 3$ hours [*Gonzalez and Tsurutani*, 1987]. For moderate storms with $Dst_{min} \leq -50$ nT, the threshold values of $B_s \geq 5$ nT and $\Delta t \geq 2$ hours were suggested [*Russell et al.*, 1974]. Moreover, *Cane et al.* [2000] found a clear correlation between B_z within ejecta or sheath regions and Dst_{min} with a correlation coefficient of 0.74. Recently, *Wu and Lepping* [2002] further confirmed the conclusion that there is a good correlation between Dst_{min} and the solar wind parameters, $(VB_z)_{min}$ and $B_{z_{min}}$, by investigating the geomagnetic activities associated with magnetic clouds on the basis of the WIND observations.

[4] To predict the *Dst* value, *Burton et al.* [1975] proposed a simple equation for the evolution of *Dst** (pressure corrected measured *Dst*) in terms of solar wind conditions: $\frac{\mathrm{d}Dst^*}{\mathrm{d}t} = Q(t) - \frac{Dst^*}{\tau}$, where Q (commonly VB_z is adopted) is the coupling function and τ is the decay time. Based on this relationship, several of models have been developed to estimate the evolution of *Dst* [e.g., *Fenrich and Luhmann*, 1998; *O'Brien and McPherron*, 2000a, 2000b; *Lundstedt et al.*, 2002]. Obviously, the last term in the above equation suggests that the loss effect becomes significant and inhibits the growth of the storm if the storm takes a long time to develop [e.g., *Daglis et al.*, 1999; *O'Brien and McPherron*, 2000a]. However, a general relationship between the intensity of the B_s field (or the $-VB_z$ electric field) and its duration Δt as a function of storm intensity Dst_{min} has not been found yet.

[5] The main aim of this letter is to find out a direct relationship between $-\overline{VB_z}$, Δt and Dst_{min} and to reveal the importance of the interplanetary parameter $-\overline{VB_z}$ and its duration Δt to the geomagnetic storms. Previous work [*Burton et al.*, 1975] suggested that the duration with $-VB_z < 0.5$ mV/m is of little geoeffectiveness. Therefore, here, Δt is the interval where $-VB_z \geq 0.5$ mV/m from the occurrence of the B_s field, which causes the storm, to the corresponding *Dst* peak, and $-\overline{VB_z}$ is the average value of $-VB_z$ during this interval. We statistically analyze 105 geomagnetic storms with $Dst_{min} \leq -50$ nT during 1998–2001. In the next section, we will introduce the selection of the sample and the method. The results are presented in Section 3. Finally, we discuss the results and give a summary.

2. Data and Method

[6] In this letter, we only select the moderate to intense storms ($Dst_{min} \leq -50$ nT), in which the contribution of interplanetary causes is prominent. Sometimes geomagnetic storms have double or triple *Dst* peaks [e.g., *Kamide et al.*, 1998; *Jordanova et al.*, 2003]. In such events, the latter *Dst* peak is produced based on the former *Dst* storm. These multiple-step storms might influence the statistical results, so we only consider the first *Dst* peak, and exclude other *Dst* peaks for the multiple-step storms. The interplanetary observations from the ACE spacecraft are used due to its relatively fixed orbit at L1 libration point. During the ACE data gap, the WIND observations are used instead.

0094-8276/03/2003GL017901$05.00

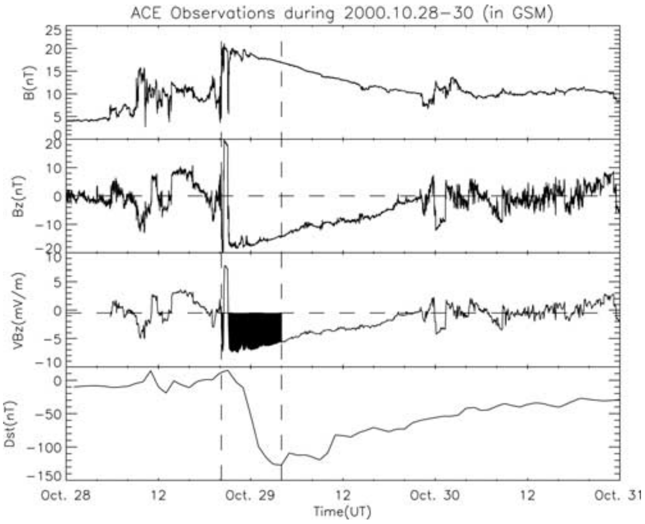

Figure 1. Observations by the ACE spacecraft from Oct. 28 to 30, 2000 (in GSM). From top to bottom are plotted: magnetic field strength B, z component field B_z, the value of VB_z, and the geomagnetic index Dst. The horizontal dashed line in the third panel denotes the value of -0.5 mV/m.

According to above criteria, a total of 105 events are selected from 1998 to 2001, among which there are 35 intense storms with $Dst_{min} \leq -100$ nT. Although the ACE spacecraft is $\sim 10^6$ km away from the magnetopause, the delay time, which is approximately less than 1 hour, is not considered, because of the 1-hour resolution of Dst. The following two examples are represented to explain the method of dealing with the data in detail.

2.1. October 29, 2000 Event

[7] Figure 1 shows the ACE observations of this event. The sudden commencement (SC) of the geomagnetic storm was at about 2000 UT on Oct. 28. Obviously, the B_s interval denoted by the vertical dashed lines should be responsible for the formation of this storm. The amplitude of the magnetic field increased suddenly from 8 nT to 20 nT approximately. B_z decreased to nearly -20 nT and formed a large B_s interval. VB_z decreased from a positive value to a negative value accordingly. Within this interval, only the durations, in which the value of $-VB_z$ was larger than 0.5 mV/m, have contribution to the storm as denoted by the filled region. Although there was still a long duration with $-VB_z \geq 0.5$ mV/m after the Dst peak, it is not included, because that $-VB_z$ duration had nothing to do with the peak. In this event, the value of $-\overline{VB_z}$ is 6.40 mV/m, and the duration Δt is 7.15 hours. Δt is shorter than the time from the beginning of the B_s interval to the Dst peak.

2.2. November 7–8, 1998 Event

[8] Figure 2 shows the ACE observations of this event. This geomagnetic storm has a double-peak structure (as marked by '1' and '2'). A long B_s interval, which caused the first Dst peak (= -81 nT) at 1700 UT, began at 1100 UT approximately on Nov. 7. In the same way, we can obtain that $-\overline{VB_z}$ = 3.57 mV/m and Δt = 5.70 hours. The second peak arrived at 0700 UT on Nov. 8. However, it was influenced by the former peak obviously because the background value of the second peak was about -50 nT, which largely deviated from the value at the quiet time. Thus, to avoid such influence, the second Dst peak is excluded from our sample though its peak value is much larger than the first.

3. Results

[9] The distributions of Δt, $-\overline{B_z}$ and $-\overline{VB_z}$ for the geomagnetic storms are shown in Figure 3, respectively. The upper row presents the situation for the storms with $Dst_{min} \leq -50$ nT. The duration Δt is not shorter than 1 hour, and 103 of 105 (98%) events are associated with $\Delta t \geq$ 2 hours. A majority (63%) of the events are concentrated in the region of 3 hours $\leq \Delta t <$ 11 hours. The peak of the distribution appears at $\Delta t \sim 6$ hours. The average value of B_s is not smaller than 3 nT, and 89 of 105 (85%) events are associated with $\overline{B_s} \geq 5$ nT. Almost all (95%) of the events are concentrated in the region of 3 nT $\leq \overline{B_s} <$ 16 nT. The $-\overline{B_z}$ distribution peak is located at about 6 nT. The value of $-\overline{VB_z}$ is larger than 1 mV/m, and 94 of all (90%) events are associated with $-\overline{VB_z} <$ 7 mV/m. The largest probability appears at $-\overline{VB_z} \sim 3$ mV/m.

[10] In our sample, 35 of 105 ($\sim$33%) events are associated with intense storms ($Dst_{min} \leq -100$ nT). The situation of such large events is similar with the former except for a small right-shift of the distributions (shown in the lower row in Figure 3). Δt is longer than 2 hours, and the peak is at about 7 hours. The value of $-\overline{B_z}$ is larger than 6 nT, and the peak appears at $\sim$10 nT. The value of $-\overline{VB_z}$ is larger than 3 mV/m, and the peak is located at 5 mV/m approximately. Obviously, all of the parameters shift toward the larger values, i.e., the more intense storms should be due to the larger $-\overline{VB_z}$ intervals.

[11] Figure 4 shows the relationship between $-\overline{VB_z}$, Δt and Dst_{min} for all the events. From the upper panel, it is found that the moderate storms are scattered over a large range from 1 hour to 28 hours, whereas the great storms with $Dst_{min} \leq -200$ nT are concentrated in a more narrow range from 2 hours to 14 hours approximately. This result implies that the longer duration is not necessary to produce the larger geomagnetic storm, because the effect of energy

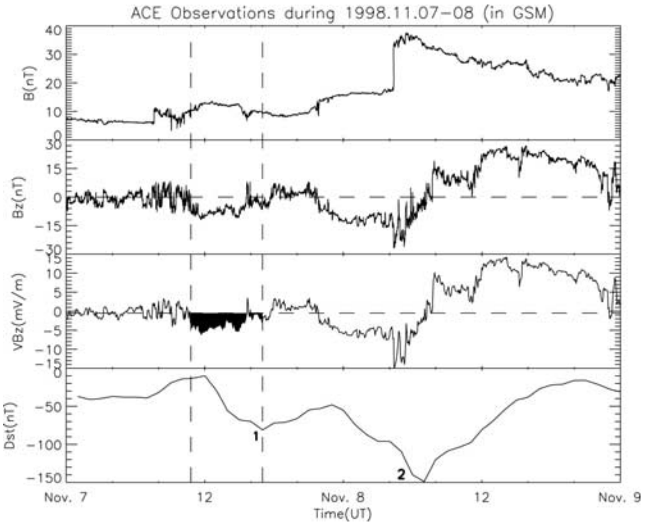

Figure 2. Observations by the ACE spacecraft from Nov. 7 to 8, 2000 (in GSM).

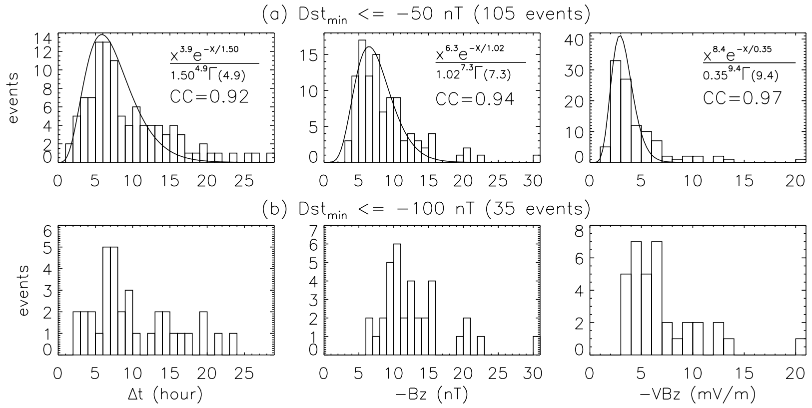

Figure 3. The Histograms showing the distributions of Δt, $-\overline{B_z}$ and $-\overline{VB_z}$ for the geomagnetic storms with $Dst_{min} \leq -50$ nT (upper) and $Dst_{min} \leq -100$ nT (lower), respectively.

loss becomes more prominent [e.g., *Daglis et al.*, 1999]. On the other hand, a good linear anti-correlation with $CC = -0.9147$ between $-\overline{VB_z}$ and Dst_{min} is presented in the second panel. The larger $-\overline{VB_z}$ is, the more intense is the geomagnetic storm.

[12] Generally, $\Phi = -\overline{VB_z}\Delta t$ is considered the magnetic flux transferred from interplanetary medium into the inner magnetosphere. However, the linear correlation ($CC = -0.7226$) between $-\overline{VB_z}\Delta t$ and Dst_{min} is weaker. Obviously, the non-linear relation between Δt and Dst_{min} weakens the correlation. The weights of $-\overline{VB_z}$ and Δt should be different. We therefore use the variable $(-\overline{VB_z})^{\alpha}(\Delta t)^{\beta}$, where α and β are tunable constants, to linearly fit the Dst data. It is found that the following empirical formula

$$Dst_{min} = -19.01 - 8.43\left(-\overline{VB_z}\right)^{1.09}(\Delta t)^{0.30}\mathrm{nT} \qquad (1)$$

has a minimum anti-correlation coefficient of −0.9528, which indicates a better goodness-of-fit, as shown in the fourth panel of Figure 4.

4. Discussion and Summary

[13] The distributions represented in the last section give the threshold values of $\overline{B_s} \geq 3$ nT and $\Delta t \geq 1$ hour for moderate geomagnetic storms with $Dst_{min} \leq -50$ nT, and the threshold values of $\overline{B_s} \geq 6$ nT and $\Delta t \geq 2$ hours for intense storms with $Dst_{min} \leq -100$ nT. In our statistical study, the investigated interval is from the occurrence of the B_s field, which causes the geomagnetic storm, to the Dst peak, and an average value of B_s is adopted. Hence, these threshold values are all different from, actually smaller than, the results obtained by *Gonzalez and Tsurutani* [1987] and *Russell et al.* [1974], in whose works the entire B_s interval and the maximum value of B_s were used. In addition, the distributions for all the storms seem to follow the Γ distribution ($CC > 0.90$) as shown in Figure 3. As for the intense storms, we do not try to fit them by Γ distribution function, because the number of the events is too small.

[14] We also obtain a good anti-correlation between Dst_{min} and $-\overline{VB_z}$ on the basis of a larger sample. The result is consistent with that obtained by *Wu and Lepping* [2002] except that the average value of $-VB_z$ is used here. *Wu and Lepping* [2002] studied the events for solar minimum (1995–1998). According to our result, such good correlation is also suitable for the ascending phase and the peak of the current solar cycle (1998–2001).

[15] Empirical formula 1 shows that the weight of Δt is less than that of $-\overline{VB_z}$. For a fixed $-\overline{VB_z}$, the value of $(Dst_{min} + 19.01) \propto (\Delta t)^{0.30}$. Thus, a long duration is not very helpful to further enhance a storm's intensity. On the contrary, a large $-\overline{VB_z}$ has a prominent contribution to creating a strong geomagnetic storm. The point 'A' labelled in Figure 4 denotes such a event associated with large $-\overline{VB_z} = 20.81$ mV/m but short $\Delta t = 4.90$ hours on March

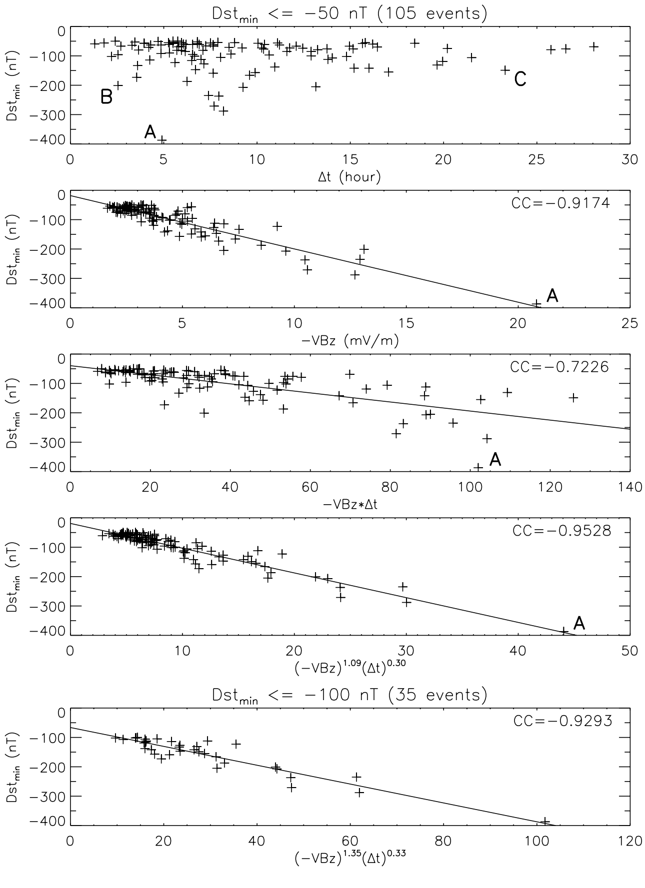

Figure 4. The scatter plots of Dst peak value versus Δt, $-\overline{VB_z}$, $-\overline{VB_z}\Delta t$, $(-\overline{VB_z})^{1.09}(\Delta t)^{0.30}$, and $(-\overline{VB_z})^{1.35}(\Delta t)^{0.33}$, respectively. The solid lines are the linear fitting curves.

31, 2001, which produced the largest geomagnetic storm with $Dst_{min} = -387$ nT. This special event has been studied by *Wang et al.* [2003a] recently During this event, a multiple magnetic cloud [*Wang et al.*, 2002], in which a extraordinary large B_s field was formed due to the sub-clouds' compression, passed through the Earth.

[16] Assuming the magnetic flux $\Phi = -\overline{VB_z}\Delta t$ = constant, we can rewrite the formula 1 as $Dst_{min} = -19.01 - 8.43\Phi^{0.30}(-\overline{VB_z})^{0.79} = -19.01 - 8.43\Phi^{1.09}(\Delta t)^{-0.79}$. As is well known, a compressed B_s interval is associated with a larger $-\overline{VB_z}$ and a shorter Δt than its original state. If Δt is shortened to a half, and $-\overline{VB_z}$ enhances 1 time accordingly, the value of (Dst_{min} + 19.01) is therefore 1.73 times its original value. Thus, the compressed B_s field should have a more prominent geoeffectiveness than the original field. This is suggests that the multiple magnetic cloud and the shock compression of preexisting southward magnetic fields tend to create the larger geomagnetic storms [*Tsurutani et al.*, 1992; *Wang et al.*, 2003b]. In addition, we investigate the large $-VB_z$ intervals during 1998–2001 using the ACE observations. There were 33 large $-VB_z$ intervals with $-\overline{VB_z} \geq 5$ mV/m and $\Delta t \geq 3$ hours identified. They all caused intense geomagnetic storms ($Dst_{min} \leq -100$ nT), among which there were 9 great storms ($Dst_{min} \leq -200$ nT). Especially, almost all (8/9) of the great storms were due to compressed interplanetary structures.

[17] The same linear fit is done for the 35 large storms (as shown in the bottom panel of Figure 4). Another formula: $Dst_{min} = -66.31 - 3.21(-\overline{VB_z})^{1.35}(\Delta t)^{0.33}$ has the best correlation coefficient. It may be estimated that the error between the two formulae is approximately less than 3% when -150 nT < Dst_{min} < -500 nT. The values of α and β suggest that the weight of $-\overline{VB_z}$ relative to Δt increases, and the loss effect becomes more prominent during a large storm.

[18] To directly relate the interplanetary observations with the ground-based measurements, we use *Dst* rather than *Dst**. Actually, the solar wind pressure will influence *Dst*. So we examine the relationship by using *Dst** as well. It is found that the tunable constants α and β are 1.06 and 0.27 respectively, approaching those derived for *Dst*, and a high correlation ($CC = -0.93$) is also obtained.

[19] Actually, the idea that VB_s is more important than Δt has been impliedly presented in *Burton et al.* [1975] work. By integrating *Burton et al.* [1975] equation (also seen in Introduction), one can get a relationship: $Dst^*_{min} \approx e^{-\Delta t/\tau}\int_0^{\Delta t}\overline{VB_z}\,e^{z/\tau}\mathrm{d}z = \tau\overline{VB_z}\left(1 - e^{-\Delta t/\tau}\right)$. Using the form: $Dst_{min} = k_0 + k_1\overline{VB_z}(1 - e^{-\Delta t/\tau})$ to fit the observations, we also obtain a high correlation coefficient of 0.9490 with $k_0 = -13.62$, $k_1 = 22.25$ and $\tau = 3.10$. This formula shows that the Dst_{min} is described as a family of hyperbolas, and suggests that a B_s interval with a long duration can not further produce a much larger storm. Compared to it, formula 1 shows a simple and direct relationship between geomagnetic storm's intensity and interplanetary parameters. For the most storms, the two formulae are comparable. However, for some large storms associated with very short or very long B_s intervals, e.g., the events 'B' ($-\overline{VB_z} = 13.11$, $\Delta t = 2.55$ and $Dst_{min} = -201$) and 'C' ($-\overline{VB_z} = 5.40$, $\Delta t = 23.30$ and $Dst_{min} = -149$) marked in Figure 4, the error of *Burton et al.* [1975] formula seems to be much larger.

[20] In summary, we have statistically studied the relationship between the *Dst* peak value, $-\overline{VB_z}$ and Δt, and obtained a simple empirical formula, which is very consistent with the observations. Our analyses do not concern the interplanetary origin of the B_s interval. These results are significant in understanding of geomagnetic storms. It is suggested again that the weight of $-\overline{VB_z}$ is much larger than that of Δt. This conclusion can illuminate that the compressed southward magnetic field has a larger geoeffectiveness. However, whether and how one can use the formula to predict the evolution of *Dst* should be studied further.

[21] **Acknowledgments.** We acknowledge the use of the data from the ACE and WIND spacecraft and the *Dst* index from World Data Center. We thank the anonymous referees for the constructive comments. This work is supported by the National Natural Science Foundation of China (49834030), the State Ministry of Science and Technology of China (G2000078405), and the Chinese Academy of Sciences (KZCX2-SW-136).

References

Burton, R. K., R. L. McPherron, and C. T. Russell, An empirical relationship between interplanetary conditions and Dst, *J. Geophys. Res.*, *80*, 4204, 1975.

Cane, H. V., I. G. Richardson, and O. C. St. Cyr, Coronal mass ejections, interplanetary ejecta and geomagnetic storms, *Geophys. Res. Lett.*, *27*, 3591–3594, 2000.

Daglis, I. A., R. M. Thorne, W. Baumjohann, and S. Orsini, The terestrial ring current: Origin, formation, and decay, *Rev. Geophys.*, *37*, 407–438, 1999.

Fairfield, D. H., and L. J. Cahill, Transition region magnetic field and polar magnetic disturbances, *J. Geophys. Res.*, *71*, 155, 1966.

Fenrich, F. R., and J. G. Luhmann, Geomagnetic response to magnetic clouds of different polarity, *Geophys. Res. Lett.*, *25*, 2999–3002, 1998.

Gonzalez, W. D., and B. T. Tsurutani, Criteria of interplanetary parameters causing intense magnetic storms ($Dst < -100$ nT), *Planet. Space Sci.*, *35*, 1101, 1987.

Jordanova, V. K., L. M. Kistler, M. F. Thomsen, and C. G. Mouikis, Effects of plasma sheet variability on the fast initial ring current decay, *Geophys. Res. Lett.*, *30*(6), 1311, doi:10.1029/2002GL016576, 2003.

Kamide, Y., N. Yokoyama, W. D. Gonzalez, B. T. Tsurutani, A. Brekke, and S. Masuda, Two-step development of geomagnetic storms, *J. Geophys. Res.*, *103*, 6917, 1998.

Lundstedt, H., H. Gleisner, and P. Wintoft, Operational forecasts of the geomagnetic *dst* index, *Geophys. Res. Lett.*, *29*(24), 2181, doi:10.1029/2002GL016151, 2002.

O'Brien, T. P., and R. L. McPherron, An empirical phase space analysis of ring current dynamics: Solar wind control of injection and decay, *J. Geophys. Res.*, *105*(A4), 7707–7719, 2000a.

O'Brien, T. P., and R. L. McPherron, Forecasting the ring current index *dst* in real time, *J. Atmospheric Sol. Terrest. Phys.*, *62*, 1295–1299, 2000b.

Russell, C. T., R. L. McPherron, and R. K. Burton, On the cause of geomagnetic storms, *J. Geophys. Res.*, *79*, 1105, 1974.

Snyder, C. W., M. Neugebauer, and V. R. Rao, The solar wind velocity and its correlation with solar and geomagnetic activity, *J. Geophys. Res.*, *68*, 6361, 1963.

Tsurutani, B. T., W. D. Gonzalez, F. Tang, and Y. T. Lee, Great magnetic storms, *Geophys. Res. Lett.*, *19*, 73, 1992.

Tsurutani, B. T., W. D. Gonzalez, Y. Kamide, and J. K. Arballo (Eds.), *Magnetic storms*, *Geophys. Monogr. Ser.*, vol. 98, AGU, Washington, D. C., 1997.

Wang, Y. M., S. Wang, and P. Z. Ye, Multiple magnetic clouds in interplanetary space, *Sol. Phys.*, *211*, 333–344, 2002.

Wang, Y. M., P. Z. Ye, and S. Wang, Multiple magnetic clouds: Several examples during March–April, 2001, *J. Geophys. Res.*, in press, 2003a.

Wang, Y. M., P. Z. Ye, S. Wang, and X. H. Xue, An interplanetary cause of large geomagnetic storms: Fast forward shock overtaking preceding magnetic cloud, *Geophys. Res. Lett.*, *30*(13), 1700, doi:10.1029/2002GL016861, 2003b.

Wu, C.-C., and R. P. Lepping, Effects of magnetic clouds on the occurrence of geomagnetic storms: The first 4 years of Wind, *J. Geophys. Res.*, *107*(A10), 1314, doi:10.1029/2001JA000161, 2002.

Y. Wang, C. L. Shen, S. Wang, and P. Z. Ye, School of Earth and Space Sciences, University of Science and Technology of China, Hefei, China. (wym@mail.ustc.edu.cn)