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Dynamic Evolution of Outer Radiation Belt Electrons due to Whistler-Mode Chorus *

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Following our preceding work, we perform a further study on dynamic evolution of energetic electrons in the outer radiation belt $L = 4.5$ due to a band of whistler-mode chorus frequency distributed over a standard Gaussian spectrum. We solve the 2-D bounce-averaged Fokker-Planck equation by allowing incorporation of cross diffusion rates. Numerical results show that whistler-mode chorus can be effective in acceleration of electrons at large pitch angles, and enhance the phase space density for energies of about 1 MeV by a factor of 10^2 or above in about one day, consistent with observation of significant enhancement in flux of energetic electrons during the recovery phase of a geomagnetic storm. Moreover, neglecting cross diffusion often leads to overestimates of the phase space density evolution at large pitch angle by a factor of 5–10 after one day, with larger errors at smaller pitch angle, suggesting that cross diffusion also plays an important role in wave-particle interaction.

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Dynamics occurring in space plasmas is essentially controlled by wave-particle interaction,[1−6] particularly for space plasmas in which a field-aligned density and anisotropy distribution of energetic particle is present.[7] During the recovery phase of geomagnetic storms, the flux of energetic electrons can vary by a factor of 10–10^3 over hours to days in Earth's outer radiation belt.[8] These relativistic electrons, which can be better modeled by a typical kappa[9] or a relativistic kappa-type distributions,[10−13] can cause serious damage to orbiting satellites.[14] The variation of radiation belt energetic electrons are considered to be produced by stochastic acceleration and loss by wave-particle interactions,[15−19] ogether with the enhanced inward radial diffusion[20] or modulation[21,22] by ULF waves. In a preceding work, Zheng *et al.*[23] solved the 2-D local Fokker-Planck equation with ignoring cross diffusion rates for a band of chorus. Since energetic particles basically bounce back and forth along the field line between the mirror points, in this study, a 2-D bounce-averaged Fokker–Planck equation with incorporation of cross diffusion rates shall be adopted to obtain a more complete picture of the evolution of radiation belt electron due to whistler-mode chorus.

Whistler-mode chorus emissions are often present in the low-density region outside the plasmapause with typical frequencies between approximately $0.05|\Omega_{eq}|$ and $0.8|\Omega_{eq}|$ ($|\Omega_{eq}|$ is the equatorial electron gyrofrequency). We assume that the whistler mode chorus is field-aligned propagated and distributed over a Gaussian frequency band peaked at ω_m with half width $\delta\omega$, a lower cutoff ω_1 and an upper cutoff ω_2:

$$B_\omega^2 = \begin{cases} B_n \exp[-(\omega-\omega_m)^2/\delta\omega^2], & \omega_1 \le \omega \le \omega_2, \\ 0, & \text{otherwise}, \end{cases} \quad (1)$$

with parameter B_n determined by

$$B_n = \frac{2B_t^2}{\pi^{1/2}\delta\omega}\left[\mathrm{erf}\Big(\frac{\omega_2-\omega_m}{\delta\omega}\Big)+\mathrm{erf}\Big(\frac{\omega_m-\omega_1}{\delta\omega}\Big)\right]^{-1}. \quad (2)$$

The dispersion relation for the standard parallel whistler mode chorus can be written as

$$c^2k^2 = \omega^2 - \frac{\omega\omega_{pe}^2}{\omega-|\Omega_e|}, \quad (3)$$

where $|\Omega_e|$ and ω_{pe} are the local electron gyrofrequency and plasma frequency respectively; ω is the wave frequency, k is the wave number. Based on the previous study,[24] the following parameters are adopted to model the stormtime whistler-mode chorus at $L = 4.5$, where the peaks of the electron phase space density are observed.[17] On the dayside: $\omega_1 = 0.1|\Omega_{eq}|$, $\omega_2 = 0.3|\Omega_{eq}|$, $\delta\omega = 0.1|\Omega_{eq}|$, $\omega_m = 0.2|\Omega_{eq}|$. We choose the wave amplitudes as $B_t = 10^{0.75+0.04\lambda}$[pT] within $\lambda \le 35°$ and the equatorial $\omega_{pe}/|\Omega_e|$ is taken 4.6. On the nightside: $\omega_1 = 0.05|\Omega_{eq}|$, $\omega_2 = 0.65|\Omega_{eq}|$, $\delta\omega = 0.15|\Omega_{eq}|$, $\omega_m = 0.35|\Omega_{eq}|$. We also assume a constant wave amplitude $B_t = 50$[pT] distributed over a latitude range $\lambda \le 15°$ and the equatorial $\omega_{pe}/|\Omega_e|$ is taken 3.8.

The 2-D bounce-averaged Fokker-Planck Equation can be expressed by[25]

$$\begin{aligned}\frac{\partial f}{\partial t} &= \frac{1}{G}\frac{\partial}{\partial \alpha_e}\left[G\Big(\langle D_{\alpha\alpha}\rangle\frac{\partial f}{\partial \alpha_e}+\langle D_{\alpha p}\rangle\frac{\partial f}{\partial p}\Big)\right]\\ &\quad+\frac{1}{G}\frac{\partial}{\partial p}\left[G\Big(\langle D_{p\alpha}\rangle\frac{\partial f}{\partial \alpha_e}+\langle D_{pp}\rangle\frac{\partial f}{\partial p}\Big)\right], \end{aligned}\quad (4)$$

where α_e denotes the equatorial pitch angle; p is the electron momentum scaled by m_e; $G = p^2T(\alpha_e)\sin\alpha_e\cos\alpha_e$ with $T \approx 1.30$–$0.56\sin\alpha_e$;[26]

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$\langle D_{\alpha\alpha}\rangle$, $\langle D_{pp}\rangle$, and $\langle D_{\alpha p}\rangle = \langle D_{p\alpha}\rangle$ stand for bounce-averaged diffusion coefficients in pitch angle, momentum and cross pitch-angle-momentum, respectively. For a dipolar geomagnetic field model, those diffusion coefficients are derived by[25]

$$\langle D_{\alpha\alpha}\rangle = \frac{1}{T}\int_0^{\lambda_m} D_{\alpha\alpha}\frac{\cos\alpha}{\cos^2\alpha_e}\cos^7\lambda d\lambda, \tag{5}$$

$$\langle D_{pp}\rangle = \frac{1}{T}\int_0^{\lambda_m} D_{pp}\frac{(1+3\sin^2\lambda)^{1/2}}{\cos\alpha}\cos\lambda d\lambda, \tag{6}$$

$$\langle D_{\alpha p}\rangle = \frac{1}{T}\int_0^{\lambda_m} D_{\alpha p}\frac{(1+3\sin^2\lambda)^{1/4}}{\cos\alpha_e}\cos^4\lambda d\lambda, \tag{7}$$

where λ is the geomagnetic latitude, λ_m is the mirror point latitude; $D_{\alpha\alpha}$, D_{pp} and $D_{\alpha p}$ are local diffusion coefficients given by[27]

$$D_{\alpha\alpha} = \frac{|\Omega_e|^2}{p^2}\Big(\frac{p^2}{\gamma^2}I_0 - 2\cos\alpha\frac{cp}{\gamma}I_1 + \cos^2\alpha c^2 I_2\Big), \tag{8}$$

$$D_{pp} = c^2|\Omega_e|^2\sin^2\alpha I_2, \tag{9}$$

$$D_{\alpha p} = -c|\Omega_e|^2\sin\alpha\Big(\frac{I_1}{\gamma} - \frac{c\cos\alpha}{p}I_2\Big), \tag{10}$$

$$I_n = \pi\sum_{\omega_r}\Big\{\frac{B_\omega^2}{B_0^2(\lambda)}\Big(\frac{\omega_r}{ck_r}\Big)^n\Big|1-\cos\alpha\frac{p}{\gamma}\frac{dk}{d\omega}\Big|^{-1}_{\omega=\omega_r}\Big\}, \tag{11}$$

where $n = 0, 1, 2$, γ is the Lorentz factor, ω_r (or k_r) is solution of the condition for electrons (with parallel velocity $v_\parallel$) in gyroresonance with parallel propagated whistler mode chorus: $\omega - v_\parallel k = |\Omega_e|/\gamma$, B_0 is the ambient magnetic field strength for a dipolar geomagnetic field model:

$$B_0(\lambda) = 3.12\times 10^4\frac{(1+3\sin^2\lambda)^{1/2}}{L^3\cos^6\lambda}\,\mathrm{nT}. \tag{12}$$

Since the whistler mode chorus are often found over a broad range of local times (2200–1300 MLT),[28] the weighted diffusion coefficients can be obtained by applying 25% drift averaging for both dayside and nightside. Figure 1 shows the weighted diffusion rates of pitch-angle $\langle D_{\alpha\alpha}\rangle$, momentum $\langle D_{pp}\rangle/p^2$ and cross pitch-angle-momentum $|\langle D_{\alpha p}\rangle|/p$. The cross diffusion coefficient becomes negative at larger energies and larger pitch-angle (above the white line). The corresponding profiles of diffusion coefficients at different indicated energies are also shown in Fig. 2. Pitch angle and cross diffusion coefficients are found to be about 10 and 3 times (or above) respectively higher than momentum diffusion coefficients at about 1 MeV, suggesting that cross terms should play an important role in wave particle interaction.

Using the parameters above, we evaluate the temporal evolution of electron phase space density (PSD) due to whistler-mode chorus by solving the 2-D bounce-averaged Fokker-Plank equation (4). In general, the cross diffusion coefficients can change very rapidly and become positive or negative, often leading to numerical problems, e.g., stability, when solving the Fokker-Planck equation by the standard finite difference method. To avoid this problem, previous works adopted either a variable transformation technique[29] or a Monte Carlo method[30] to solve the bounce averaged Fokker-Plank equation with cross diffusion. We adopt a split operator technique,[31] a fully implicit scheme for the diagonal diffusion, and a two step alternative direction implicit scheme[32] for the cross diffusion to solve the Fokker-Planck equation. The numerical grid sets to be 101×101 and uniform in pitch angle and nature logarithmic in momentum. The time step should be small enough to prevent the numerical instability, and in our simulation the time step is set to be 5 s.

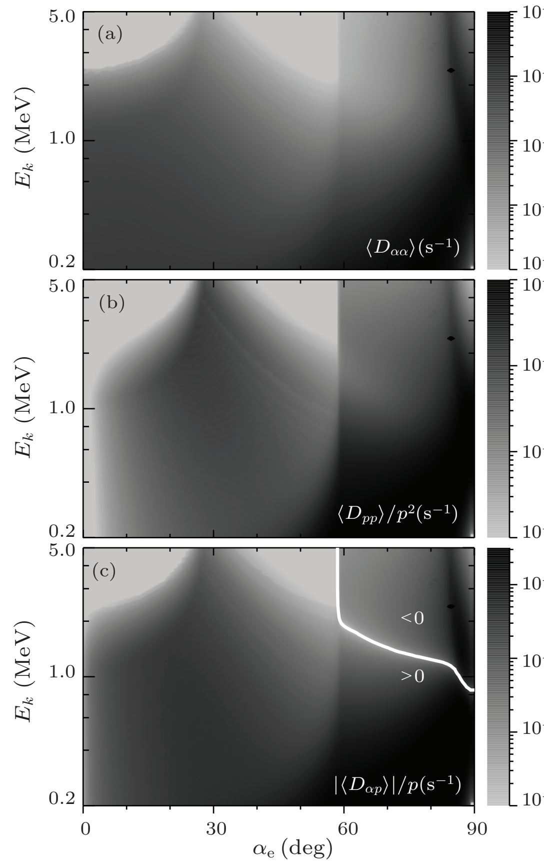

Fig. 1. Two-dimensional pitch angle diffusion rate (a), momentum diffusion rate (b) and cross diffusion rate (c). The white line denotes the boundary above (or below) which cross diffusion rate is smaller (or larger) than zero.

Following previous work,[29] the initial distribution of radiation belt electron is taken $f|_{t=0} = \exp[-(E-0.2)/0.1]\sin\alpha_e/p^2$, with kinetic energy $E = 0.511[(1+p^2/c^2)^{1/2} - 1]$ MeV. The value of f is fixed at lower boundary ($E = 0.2$ MeV) to simulate a balance between losses to the atmosphere and continuous convective injection of plasma sheet electrons, while $f = 0$ is imposed at the upper boundary ($E = 5.0$ MeV); f is also assumed to be zero at the loss-cone $\alpha_e = \alpha_L$ ($\sin\alpha_L = L^{-3/2}(4-3/L)^{-1/4}$), and $\partial f/\partial\alpha_e = 0$ is taken as the boundary condition at $\alpha_e = 90^\circ$.

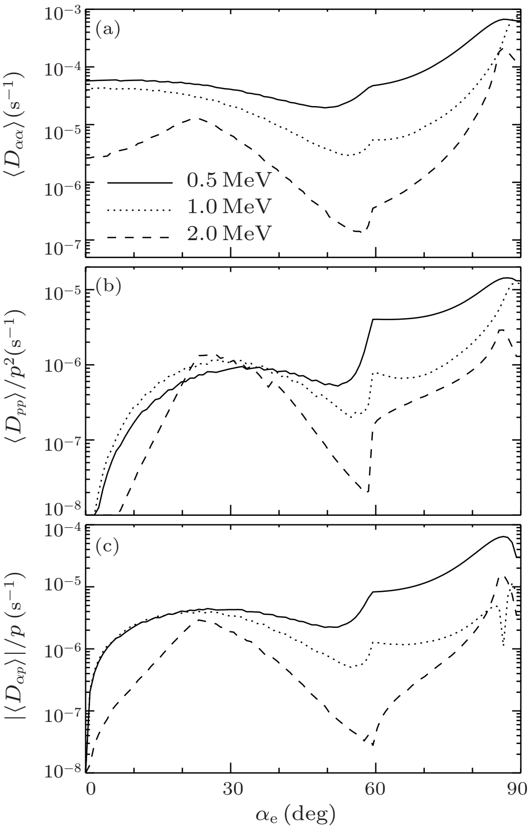

Fig. 2. Pitch-angle diffusion rate (a), momentum diffusion rate (b) and cross diffusion rate (c) for different indicated energies, corresponding to Fig. 1.

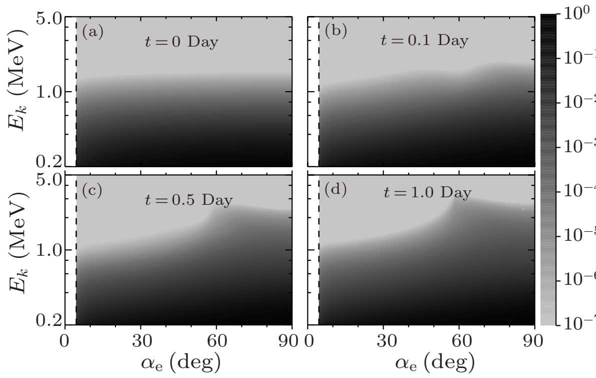

Fig. 3. Evolution of PSD due to interaction with chorus after (a) 0, (b) 0.1, (c) 0.5 and (d) 1 day with cross diffusion rates. The vertical dashed lines correspond to the loss-cone $\alpha_L \approx 4.4°$.

The evolution of PSD as functions of pitch angle and kinetic energy are shown in Fig. 2. Clearly, PSD enhancement is found to occur primarily at higher energies (~ 0.5 MeV and above) and higher pitch angles (about 60° and above), indicating that chorus is responsible for accelerating energetic electrons trapped in the radiation belts. To investigate the effect of cross diffusion rates, the bounce-averaged Fokker-Planck equation is also solved without cross terms, and the corresponding results are shown in Fig. 1. It is demonstrated that evolutions of PSD without cross diffusion rates are basically higher than those with the cross diffusion rates especially at small pitch angle. Figure 3 presents PSD evolution of electrons with energies 1.0 MeV and 2.0 MeV after different times with and without cross diffusion rates. Obviously, the PSDs for energies of about 1.0 MeV are found to increase by a factor 10^2 or above with the cross diffusion during one day. The timescale is comparable to the observed timescale for flux j (since $j = p^2 f$) increasing in the radiation belts during the recovery phase of magnetic storms. Meanwhile, neglecting cross diffusion results in overestimates of the phase space density evolution by a factor of 5–10 after one day for the specified wave modes at large pitch angle, with larger errors at smaller pitch angle, suggesting that cross diffusion rates also play important role in wave-particle interaction. It should be pointed out that wave power and the ratio of electron plasma frequency to electron gyrofrequency f_{pe}/f_{ce} are very critical to the efficiency of wave acceleration. Since all diffusion coefficients have the same ratio to wave power B_ω^2 (see Eqs. (8)–(11)), the overestimate of PSD due to ignoring cross diffusion should occur for any wave power. In addition, as wave power increases, the time scale for acceleration reduces. In addition, whistler mode acceleration is more efficient in regions of low density, since this increases the phase velocity of the waves for the dominant cyclotron resonance.[33] Observation show that whistler mode wave power and f_{pe}/f_{ce} vary considerably with magnetic activity, L, magnetic local time MLT, and magnetic latitude λ,[28] and future works will be presented to take these variations into account.

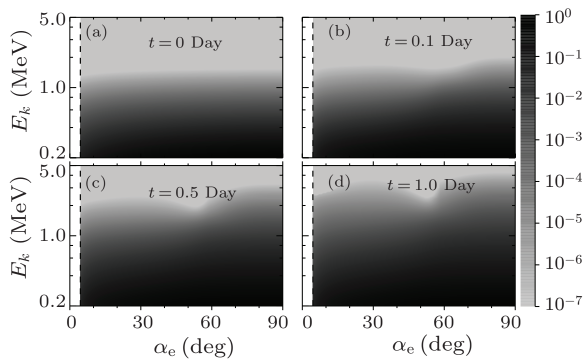

Fig. 4. The same as Fig. 3 but without cross diffusion.

In summary, we have evaluated dynamic evolution of energetic electrons in the outer radiation belt $L = 4.5$ due to whistler-mode chorus. We carry out the detailed calculation of all the bounce-averaged diffusion coefficients. We show that chorus can produce substantial acceleration of electrons at large pitch angles, and enhance the phase space density for energies of about 1 MeV by a factor of 10^2 or above in about one day, consistent with observation and previous numerical results (e.g., Refs. [29,30]). Moreover, numerical check shows that neglecting cross diffusion rates can yield overestimates of the phase space density evolution at large pitch angle by a factor of 5–10 after one day for the specified wave modes, with larger errors at smaller pitch angle, indicating that cross diffusion rates are also crucial in controlling wave-particle interaction. The current simulation results for acceleration at $L = 4.5$ should be applied to other L-shell with the wave power data available.

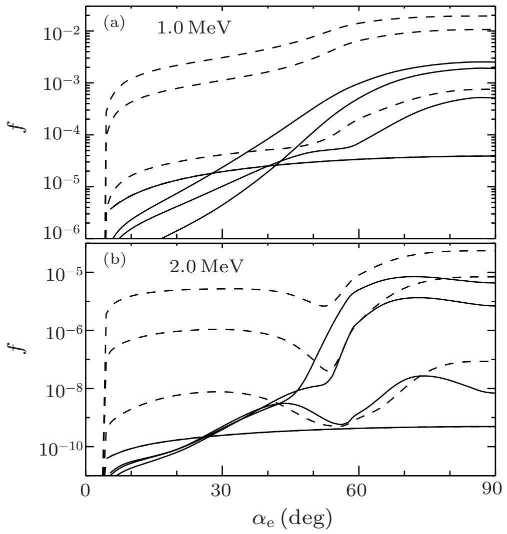

Fig. 5. Evolution of PSD for different indicated kinetic energies 1.0 MeV (a) and 2.0 MeV (b) after 0, 0.1, 0.5 and 1 day. The solid and dashed lines represent the results with and without cross diffusion.

Future modeling efforts should be directed toward consideration of each important process including various wave-particle interaction and improving evaluation of diffusion rates (possibly oblique waves rather than field-aligned waves). Furthermore, the present code can be directly extended to the 3D case with incorporation of radial diffusion to simulate the global dynamics of energetic electrons in the radiation belts.

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