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Pitch Angle Distribution Evolution of Energetic Electrons by Whistler-Mode Chorus *

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We develop a two-dimensional momentum and pitch angle code to solve the typical Fokker–Planck equation which governs wave–particle interaction in space plasmas. We carry out detailed calculations of momentum and pitch angle diffusion coefficients, and temporal evolution of pitch angle distribution for a band of chorus frequency distributed over a standard Gaussian spectrum particularly in the heart of the Earth's radiation belt $L = 4.5$, where peaks of the electron phase space density are observed. We find that the Whistler-mode chorus can produce significant acceleration of electrons at large pitch angles, and can enhance the phase space density for energies of $0.5 \sim 1$ MeV by a factor of 10 or above after about 24 h. This result can account for observation of significant enhancement in flux of energetic electrons during the recovery phase of a geomagnetic storm.

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Cyclotron wave–particle interaction plays a crucial role in dynamics occurring in space plasmas,[1−6] e.g., primarily responsible for stochastic acceleration and pitch angle scattering of energetic particles in the Earth's radiation belts.[7,8] During the recovery phase of magnetic storms, the flux of energetic electrons can vary by a factor of 10–10^3 over hours to days in the Earth's outer radiation belt.[9] This flux enhancements are considered to be associated with the acceleration and loss processes by wave–particle interactions occurring in the Earth's magnetosphere.[10−13] Since energetic electrons pose a serious hazard to geostationary orbiting satellites,[14] it is very important to obtain understanding of acceleration and loss processes in order to analyse and predict the Earth's radiation environment. A number of mechanisms were suggested to account for the acceleration: including shock acceleration associated with rapid flux enhancement on timescales of minutes, inward radial diffusion associated with enhanced ULF waves,[15,16] and in situ acceleration particularly by Whistler mode waves through Doppler-shifted cyclotron resonance[17] since observation shows that peaks in the electron phase space density[18] and pitch angle distributions[19] occur in the outer radiation belt near $L = 4.5$. However, previous work[20] solved 1-D momentum (or energy) Fokker–Planck equation by assuming an isotropic or quasi-isotropic distribution to study stochastic acceleration of electrons due to electromagnetic waves. Furthermore, under certain magnetospheric conditions (e.g, for lower-band chorus), the cyclotron resonant energies can approach or exceed the electron rest energy $m_e c^2$,[1] energetic particles should be modelled by a typical kappa[21] or a relativistic kappa-type distribution.[22,23] Hence, in order to better understand the acceleration mechanism a fully relativistic treatment is required, e.g., a field-aligned density and anisotropy distribution of energetic particle.[24] In this study, we develop a 2-D momentum/pitch-angle code to solve a relativistic diffusion equation which controls wave–particle interaction by adopting a recently introduced relativistic kappa-type (KT) distribution.[25]

The dispersion relation for the standard parallel Whistler mode chorus can be written[1]

$$c^2k^2 = \omega^2 - \frac{\omega\omega_{pe}^2}{\omega - |\Omega_e|}, \tag{1}$$

where $|\Omega_e|$ and ω_{pe} are the electron gyrofrequency and plasma frequency respectively; ω is the wave frequency, k is the wave number. In general, one standard way[20] for modelling the spectral energy density of Whistler-mode chorus is to adopt a Kolmogorov spectrum, i.e. $B^2(k) = Ak^{-\nu}$ ($k_1 \leq |k| \leq k_2$). However, this puts more power at low wave numbers and hence at low frequencies, not always consistent with observation. Since Whistler mode chorus waves have been found to occur over a finite frequency band, another typical method[17,19] is to assume that the Whistler mode chorus is distributed over a Gaussian frequency band peaked at $\omega = \omega_m$ and with half width $\delta\omega$

$$B_\omega^2 = \begin{cases} B_n \exp\Big[-\dfrac{(\omega-\omega_m)^2}{\delta\omega^2}\Big], & \text{for } \omega_1 \leq \omega \leq \omega_2, \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

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with parameter B_n determined by

$$B_n = \frac{2B_t^2}{\pi^{1/2}\delta\omega}\Big[\mathrm{erf}\Big(\frac{\omega_2-\omega_m}{\delta\omega}\Big)+\mathrm{erf}\Big(\frac{\omega_m-\omega_1}{\delta\omega}\Big)\Big]^{-1}, \tag{3}$$

where B_t represents the wave magnetic field strength.

In the kinetic theory of wave–electron interaction in a relativistic plasma, the general resonance condition for parallel Whistler mode waves obeys

$$\omega - kv_{\parallel} = |\Omega_e|/\gamma, \tag{4}$$

where $\gamma=[1+p^2/c^2]^{1/2}$ is the Lorentz factor, p is the electron momentum scaled by m_e and c is the speed of light in vacuum.

The 2D Fokker–Planck equation can be expressed by[26]

$$\frac{\partial f}{\partial t} = \frac{1}{\sin\alpha}\frac{\partial}{\partial\alpha}\Big(D_{\alpha\alpha}\sin\alpha\frac{\partial f}{\partial\alpha}\Big)+\frac{1}{p^2}\frac{\partial}{\partial p}\Big(p^2D_{pp}\frac{\partial f}{\partial p}\Big), \tag{5}$$

where α denotes the pitch angle; $D_{\alpha\alpha}$ and D_{pp} stand for the diffusion coefficients of pitch angle and momentum, and are given by[26]

$$D_{\alpha\alpha} = \frac{|\Omega_e|^2}{p^2}\Big(\frac{p^2}{\gamma^2}I_0 - 2\cos\alpha\frac{cp}{\gamma}I_1+\cos^2\alpha c^2I_2\Big), \tag{6}$$

$$D_{pp} = c^2|\Omega_e|^2\sin^2\alpha I_2, \tag{7}$$

$$I_n = \pi\sum_{\omega_r}\Big\{\frac{B_\omega^2}{B_0^2}\Big(\frac{\omega_r}{ck_r}\Big)^n\Big|1-\cos\alpha\frac{p}{\gamma}\frac{dk}{d\omega}\Big|^{-1}_{\omega=\omega_r}\Big\}, \tag{8}$$

where $n=0,1,2$, ω_r (or k_r) is the solution of resonant equation (4) together with the wave dispersion relation (1), B_0 is the equatorial ambient magnetic field strength with $B_0=3.12\times10^4/L^3\,\mathrm{nT}$ for a dipolar geomagnetic field model.

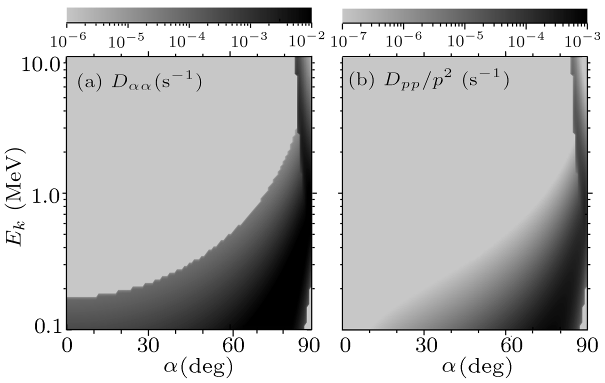

Fig. 1. Two-dimensional pitch angle diffusion coefficient (a) and momentum diffusion coefficient (b).

We choose boundary conditions for the pitch angle operator as $f=0$ at the loss-cone $\alpha=\alpha_L$ ($\sin\alpha_L = L^{-3/2}(4-3/L)^{-1/4}$) and $\partial f/\partial\alpha = 0$ at $\alpha=90$. For the energy diffusion operator we set $f=\mathrm{const}$ at the lower boundary 0.1 MeV to simulate a balance between losses to the atmosphere and continuous convective injection of plasma sheet electrons, and $f=\mathrm{const}$ at the upper boundary 10 MeV.[27]

In general, energetic particles existing in planetary magnetospheres and other plasmas often display a power-law and can be well modelled by a typical kappa distribution. However, the kappa distribution satisfies the form: $\propto[1/v^2]^{(\kappa+1)}$ instead of $\propto[1/p]^{\kappa+1}$ at the relativistic energy, appearing to be inconsistent with the power-law since the relativistic energy is proportional to p instead of v^2. Recently, Xiao *et al.*[23,28] have adopted a relativistic KT distribution[25] to fit solar energetic particle spectra observed by the IMP 8 and Helios 1 and 2 spacecraft, and energetic electrons spectrum observed by the SOPA instrument on board the 1989-046 and LANL-01A satellites at geosynchronous orbit. It is found that the relativistic kappa-type distribution fits well with the observed data during different universal times in both the lower and higher energies.

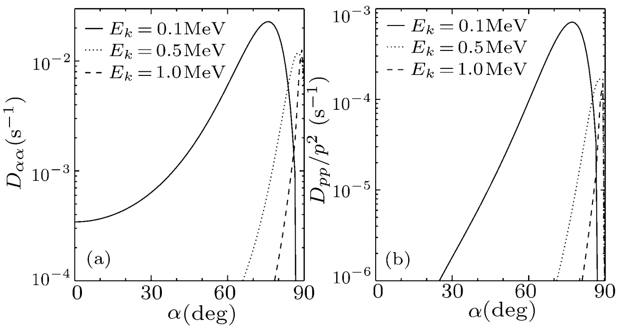

Fig. 2. Pitch angle diffusion coefficient (a) and momentum diffusion coefficient (b) for different indicated energies.

Since the resonant energies can approach $\sim$MeV, we assume that at $t=0$, the space density f takes a recently introduced relativistic KT distribution for $\alpha>\alpha_L$ [25]

$$f(p,\alpha) = \frac{1}{2\pi^{3/2}}\frac{\Gamma((q+3)/2)}{\Gamma((q+2)/2)}\frac{1}{I} \cdot\Big[1+\frac{\sqrt{1+p^2/c^2}-1}{\kappa\theta^2}\Big]^{-(\kappa+1)}\sin^q\alpha, \tag{9}$$

where q is the loss-cone index, θ^2 is the effective thermal energy scaled by m_ec^2, κ is the spectral index, Γ is the gamma function, and I is a normalized constant given by

$$I = \frac{8B(3/2,\kappa-2)}{2\kappa-1}\Big\{3F\Big(\kappa+1;\frac{5}{2};\kappa+\frac{1}{2};1-\frac{2}{\kappa\theta^2}\Big) + (\kappa-2)F\Big(\kappa+1;\frac{3}{2};\kappa+\frac{1}{2};1-\frac{2}{\kappa\theta^2}\Big)\Big\}, \tag{10}$$

where F is the hypergeometric function and B is the beta function. This new KT distribution, which follows the power-law not only at the lower energies but also at the relativistic energies, is found to show different effect from the regular kappa distribution on the Whistler-mode instability.[29]

Whistler-mode chorus emissions are excited in the low-density region outside the plasmapause by the injection of plasma sheet electrons into the inner magnetosphere during enhanced storm time convection. Chorus emissions generally occur in discrete microbursts at frequencies between approximately $0.2|\Omega_e|$ and $0.8|\Omega_e|$ of the equatorial electron gyrofrequency. Based on the previous work,[28] at $L \approx 4.5$, we choose the following parameters $B_t = 0.1\,[\mathrm{nT}]$, $\omega_1 = 0.05|\Omega_e|$, $\omega_2 = 0.65|\Omega_e|$, $\delta\omega = (\omega_2 - \omega_1)/4$, $\omega_m = (\omega_2 + \omega_1)/2$. The background density is taken $N_b = 124(3/L)^4\,\mathrm{cm}^{-3}$.[30]

In Fig. 1, we plot pitch angle and momentum diffusion rates as functions of pitch angle α and kinetic energy for $L = 4.5$. Pitch angle and momentum diffusion coefficients as a function of pitch angle α for different indicated energies are shown in Fig. 2. Both pitch angle and momentum diffusion rates are found to be large at lower energies and large pitch angles, suggesting that acceleration of electrons due to chorus basically occurs in high pitch angles.

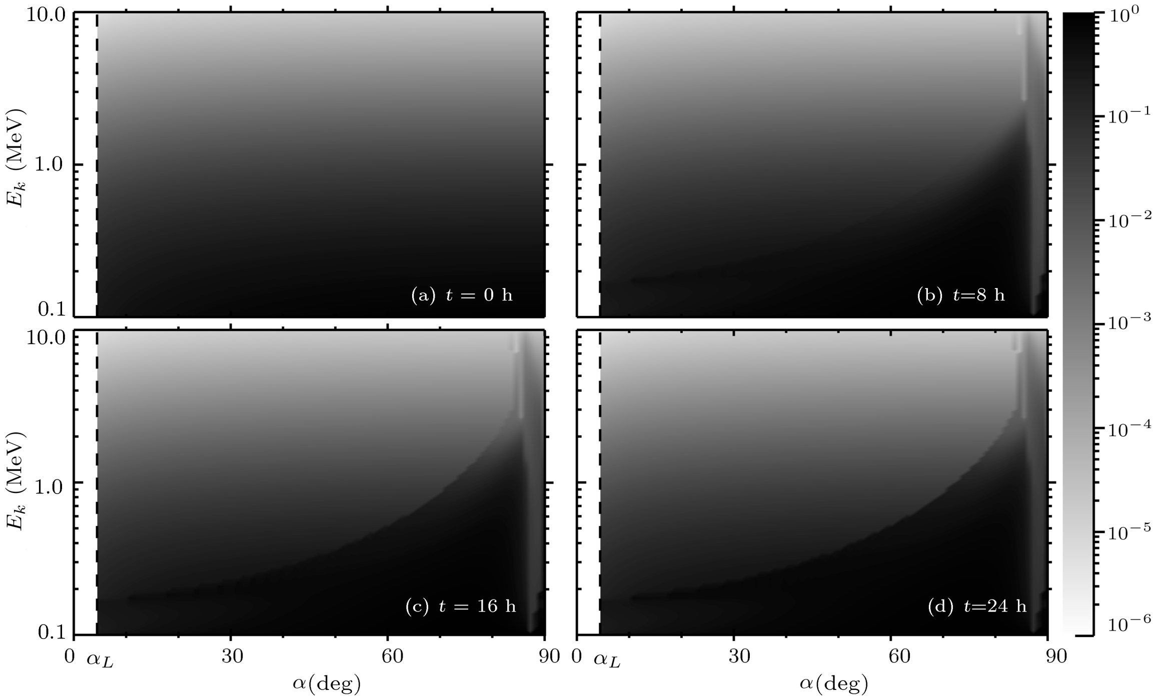

Fig. 3. (a) Initial PAD of electrons with various pitch angles and kinetic energies. Evolution of PAD due to interaction with chorus after 8 h (b), 16 h (c) and 24 h (d). The vertical white dotted lines correspond to the loss-cone $\alpha_L \approx 4.4°$.

Using the parameters above, we solve the 2D Fokker–Plank equation (5) to obtain the temporal evolution of electron pitch angle distribution (PAD) interacting with Whistler-mode chorus. We implement the numerical algorithm by adopting a split operator technique and an unconditionally stable, implicit numerical scheme. The numerical grid sets to be 101×101 and uniform in pitch angle and logarithmic in momentum. In Fig. 3, we present the initial PAD, and evolution of PAD as functions of pitch angle and kinetic energy due to interaction with chorus after different indicated times. It is shown that there is an increase in PAD for high-energy electrons (particularly $\sim 0.5\,\mathrm{MeV}$ and above) and a decrease at lower energies, implying that during wave–particle interaction, lower energy electrons transfer energy to wave while high energy electrons gain energy from wave. The result further supports previous work that wave amplification is basically related with pitch-angle scattering to smaller pitch-angles and a net loss of electron energy; while wave damping is associated with pitch-angle scattering to larger pitch-angles and electron. Figure 4 shows initial PAD and evolution of PAD with various pitch angles for different indicated kinetic energies and different indicated times. The space density $f(> 0.5\,\mathrm{MeV})$ is shown to increase by more than an order of magnitude after about $\sim 24\,\mathrm{h}$ at high pitch angles $\sim 60°$ and above. These timescales are comparable to the observed timescale for flux j(since $j = p^2 f$) increasing in the radiation belts during the recovery phase of magnetic storms.

In summary, we have evaluated PAD of energetic electrons due to Whistler mode chorus with a standard Gaussian spectrum distribution near $L = 4.5$, where peaks in the phase space density often occur. We use a 2D momentum and pitch angle code to solve the typical Fokker–Planck equation associated with wave–particle interaction in space plasmas. It is demonstrated that Whistler-mode chorus has substantial potential for acceleration of electrons at large pitch angles, and can increase the phase space density at energies of $0.5\sim 1\,\mathrm{MeV}$ or above by more than a factor of 10 in about 24 h. These results present further understanding for observation of significant enhancement in flux of energetic electrons during the recovery phase of a geomagnetic storm.

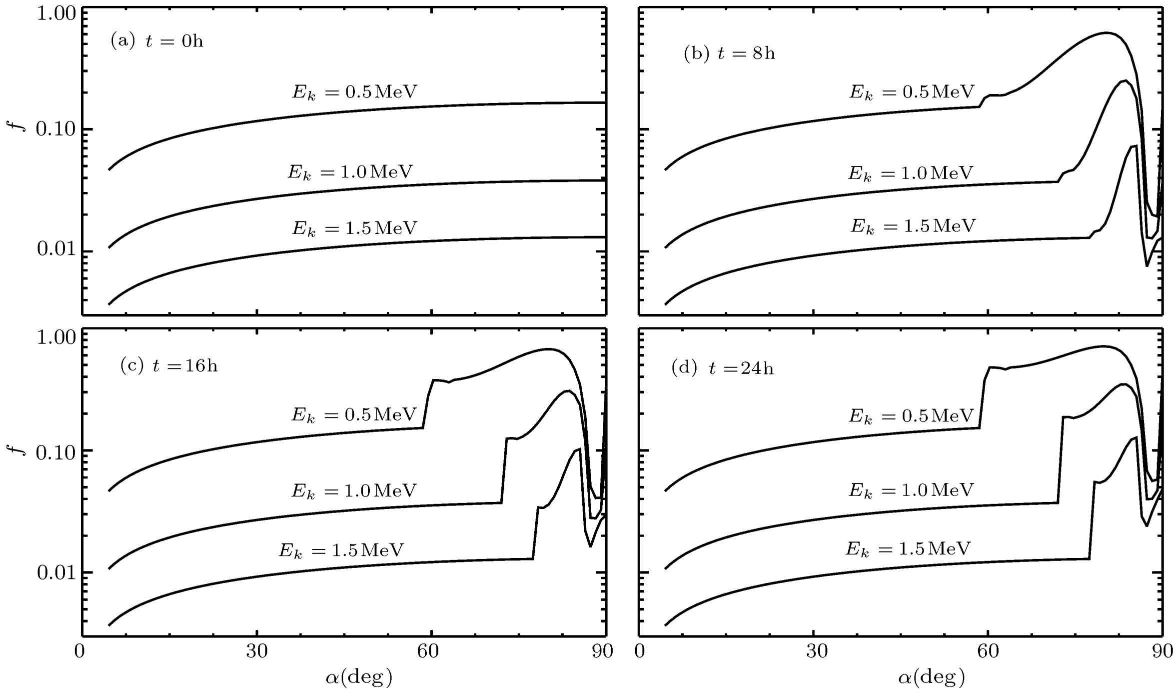

Fig. 4. Initial PAD (a) and evolution of PAD with various pitch angles after 8 h (b), 16 h (c) and 24 h (d) for different indicated kinetic energies.

It is well-known that cross diffusion coefficients also control energization and pitch angle scattering of energetic particles, and energetic particles basically bounce back and forth along the field line between the mirror points. In order to present a more complete picture of PAD evolution of energetic particles, further work is required to evaluate a bounce-averaged Fokker–Planck equation with incorporation of the cross-diffusion coefficients.

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