# Heating of Ions by Alfvén Waves via Nonresonant Interactions

C. B. Wang,[1,*] C. S. Wu,[1,2,†] and P. H. Yoon[2,‡]

[1]CAS Key Laboratory of Basic Plasma Physics, School of Earth and Space Sciences, University of Science and Technology of China, Hefei, Anhui 230026, China

[2]Institute for Physical Science and Technology, University of Maryland, College Park, Maryland 20742, USA

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Finite-amplitude intrinsic Alfvén waves exist pervasively in astrophysical and solar-terrestrial environment. It is generally believed that linear wave-particle resonant interaction between thermal protons and Alfvén waves is ineffective when the proton beta is low. However, this Letter demonstrates that the ions can be heated by Alfvén waves via nonresonant nonlinear interaction. Contrary to the customary expectation, it is found that the lower the plasma beta value, the more effective is the heating process. It is also shown that the ion temperature increase is more prominent along perpendicular direction.

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It is well known that enhanced Alfvén waves exist pervasively in the solar wind [1–5]. The long-standing problems have to do with understanding and characterizing the source and the influence of these waves on the solar-terrestrial plasma. For details, see the review articles [6–13]. One of the fundamental outstanding questions is whether Alfvén waves can lead to plasma heating. According to linear theory, Alfvén waves propagating along an ambient magnetic field can only interact with charged particles via cyclotron resonance $\omega \pm \Omega_q - kv_\parallel = 0$, where $\omega$ and $k$ are the wave frequency and wave number; $\Omega_q$ denotes the gyrofrequency for species labeled $q$; $v_\parallel$ is the velocity component parallel to the ambient magnetic field; and $\pm$ designates right- and left-hand polarization. The effect of Doppler shift on the Alfvén wave frequency is small if the thermal protons have a beta value much less than unity. As a result, there is no way the foregoing resonance condition can be satisfied. For this reason, it is generally believed that effective interactions between protons and Alfvén waves are unlikely.

However, linear resonance condition is valid only if the wave amplitude is vanishingly low. But in reality, *in situ* observations find that in interplanetary space the Alfvén wave magnetic field amplitude $\delta B_w$ is often comparable to the ambient magnetic field intensity $B_0$ such that it is common to find $\delta B_w/B_0$ on the order of 0.3–0.5 [2]. In some case it may even reach 0.8 [14].

In two recent publications [15,16] Chen and his collaborators show that a large-amplitude Alfvén wave propagating in an oblique direction can lead to subharmonic cyclotron resonance when the wave amplitude is sufficiently large. They demonstrate that the ion orbit becomes chaotic under certain circumstances, implying that an ensemble of ions may undergo heating under such a situation. This finding is interesting since heating of homogeneous plasmas by an Alfvén wave has never been discussed before. Intuitively one would expect that the magnetic moment of an ion is conserved in the presence of an ultra-low frequency wave. References [15,16] show that nonlinear effects can alter the basic physical picture.

In present study emphasis is placed on a nonresonant pitch-angle scattering process which can randomize the ion motion by a spectrum of Alfvén waves. However, this heating process is due to a random spatial velocity distribution, as will be explained later. Moreover, this process is only effective for low-beta plasmas.

Let us proceed with an analytic theory. Consider that intrinsic Alfvén waves are propagating along the ambient magnetic field, $\mathbf{B}_0 = B_0\mathbf{i}_z$. Without loss of generality, let us consider right-hand circular polarization. The wave magnetic field vector can thus be expressed as

$$\delta\mathbf{B}_w = \sum_k B_k[\cos\phi_k\mathbf{i}_x + \sin\phi_k\mathbf{i}_y], \tag{1}$$

where $\phi_k = kv_At - kz + \varphi_k$ is the wave phase, $\varphi_k$ being the phase constant, $\omega = kv_A$ is the wave frequency, and $\mathbf{i}_x$ and $\mathbf{i}_y$ are unit vectors. The corresponding wave electric field is given by $\delta\mathbf{E}_w = -(v_A/c)\mathbf{b}\times\delta\mathbf{B}_w$, where $\mathbf{b} = \mathbf{i}_z$ is a unit vector parallel to $\mathbf{B}_0$. Here we remark that the wave field $\delta\mathbf{B}_w$ in Eq. (1) is a resultant vector, which consists of a large number of discrete components with different wave numbers. In the following we pay attention to the protons only, whose equation of motion is given by

$$\frac{d\mathbf{v}}{dt} = \frac{e}{m_pc}\mathbf{v}\times(\mathbf{B}_0 + \delta\mathbf{B}_w) + \frac{e}{m_p}\delta\mathbf{E}_w, \qquad \frac{d\mathbf{r}}{dt} = \mathbf{v}. \tag{2}$$

We first rewrite Eq. (2) in terms of the quantities [17], $u_\perp = v_x + iv_y$ and $\delta B_w = \sum_k B_k e^{i\phi_k}$. This leads to

$$\begin{aligned} &\frac{du_\perp}{dt} + i\Omega u_\perp = i(v_z - v_A)\sum_k \Omega_k e^{i\phi_k}, \\ &\frac{dv_z}{dt} = -\mathrm{Im}\left(u_\perp\sum_k\Omega_k e^{-i\phi_k}\right), \qquad \frac{dz}{dt} = v_z, \end{aligned} \tag{3}$$

where $\Omega = eB_0/m_pc$ (the proton gyrofrequency) and

 

$\Omega_k = eB_k/m_p c$. We impose the initial conditions $v_z = v_z(0)$, $u_\perp = u_\perp(0)$, and $z = z(0)$. Hereafter, we consider $\Omega_k/\Omega = B_k/B_0 \ll 1$, and $|\Omega| \gg |k[v_z(0) - v_A]|$. Following Ref. [17] we obtain the following approximate solution:

$$u_\perp = u_\perp(0)e^{-i\Omega t} - [v_A - v_z(0)]\sum_k G_k,$$
$$G_k = \Omega_k e^{i[k\varphi_k - kz(0)]}\frac{e^{ik[v_A - v_z(0)]t} - e^{-i\Omega t}}{\Omega + k[v_A - v_z(0)]}. \tag{4}$$

This solution may give readers an impression that it is valid only for a short time interval. Actually Eq. (4) is derived without imposing such an assumption, as can be seen from Ref. [17] where the details are discussed. On the other hand, the test-particle calculation based on Eq. (2) does show that the time scale required for a proton to reach the time asymptotic state is indeed very short, say around a gyro period. For a "cold" proton for which $v_A \gg v_z(0)$, we obtain

$$v_\perp^2(t) = v_\perp^2(0) + v_A^2 \sum_k \sum_{k'} G_k G_{k'}^* - v_\perp(0) v_A\left(e^{-i(\alpha - \Omega t)}\sum_k G_k + e^{i(\alpha-\Omega t)}\sum_k G_k^*\right), \tag{5}$$

where $\alpha$ is the gyro-phase angle. Note that $v_\perp^2$ is enhanced by the wave fields. This is attributed to nonresonant pitch-angle scattering. The above result is applicable to a single proton. Let us now apply the above result to low-beta plasma which consists of an ensemble of protons. If the characteristic spatial dimension of our system is much larger than typical Alfvén wavelength, then we may take an ensemble average of Eq. (5) over the initial position of each particle, as well as over time and gyro-phase angle. Then we find that,

$$\frac{2T_\perp}{m_p} \equiv v_\perp^2(t) \simeq v_\perp^2(0) + 2v_A^2 \sum_k \frac{\Omega_k^2}{(\Omega + kv_A)^2}, \tag{6}$$

where $T_\perp$ is the perpendicular kinetic temperature, and $\Omega t \gg 1$ is assumed. For nondispersive Alfvén waves, the Alfvén speed represents the common phase speed of all waves with different wave numbers. Therefore, there exists a reference frame in which all waves are stationary and in this frame the energy of each particle is conserved. Making use of this condition one can show that the average value of $v_z^2(t)$ may be written as

$$\frac{T_z}{m_p} \simeq v_z^2(t) \simeq v_z^2(0) + 2v_z(0)v_A \sum_k \frac{\Omega_k^2}{(\Omega + kv_A)^2}, \tag{7}$$

where $T_z$ is the parallel kinetic temperature. Here we remark that the notion of kinetic temperatures is meaningful only when $\Omega t > \Omega t_A$, where $t_A \simeq \lambda/\sqrt{T_z/m_p}$ is the phase mixing time for the protons originated at different initial positions along the $z$ axis. If we further apply the condition $\Omega \gg kv_A$, Eqs. (6) and (7) reduce to

$$T_z = T_i\left[1 + 2\frac{v_A}{v_p}\left(\frac{\delta B_w^2}{B_0^2}\right)\right], \qquad T_\perp = T_i\left[1 + \frac{v_A^2}{v_p^2}\left(\frac{\delta B_w^2}{B_0^2}\right)\right], \tag{8}$$

respectively. In the above, we have expressed the initial velocity in terms of the initial temperature $T_i$, where we have made use of the relation, $T_i/m_p \simeq v_z^2(0) \simeq v_\perp^2/2 \equiv v_p^2$. Equation (8) shows that the kinetic temperature along $z$ direction should be much less than that in the transverse direction, while both should be enhanced by the waves.

In order to verify that the analytic theory is indeed justified, we carry out the test-particle simulation. Note that the test-particle simulation is valid in low-beta plasmas, which we assume to be. We discretize the Alfvén wave number by $k_j = k_{\min} + (j-1)(k_{\max} - k_{\min})/(J-1)$, for $j = 1, \ldots, J$, where $k_{\min} = k_1 = 0.01\ \Omega/v_A$ and $k_{\max} = k_1 = 0.05\ \Omega/v_A$. This range of wave numbers implies that we are considering $0.01\ \Omega < \omega < 0.05\ \Omega$ so that the wave frequencies are much lower than the proton gyrofrequency. The amplitude of each wave mode is considered to be constant. We have considered two values of $\delta B_w^2/B_0^2$ in the calculation, namely, $\delta B_w^2/B_0^2 \sim 0.05$ and $\sim 0.12$. The total number of test particles is $10^5$, which are randomly distributed during the time interval $\Omega t = [0, 2\pi]$ and along the spatial range $z\Omega/v_A = [0, 3000]$. Their initial velocities are assumed to have a Maxwellian distribu-

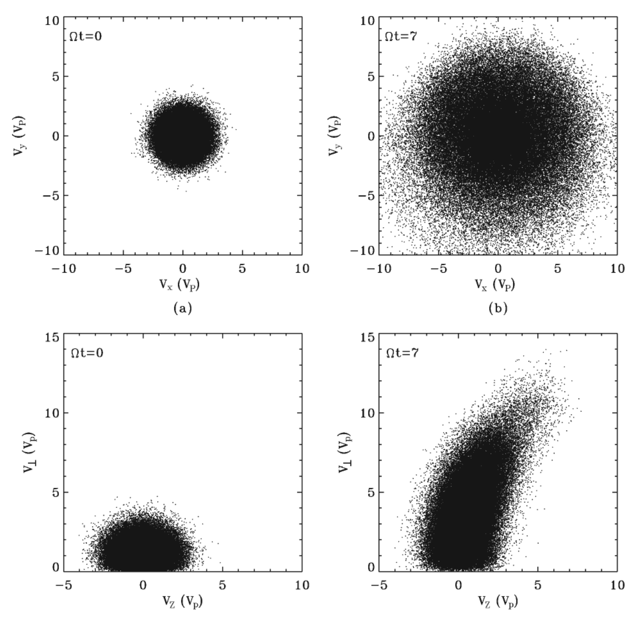


FIG. 1. Velocity scatter plots of the test particles in the $v_x$-$v_y$ [(a), (b)] and $v_\perp$-$v_z$ [(c), (d)] spaces at time $\Omega t = 0$ and $\Omega t = 7$, for input parameters $(\delta B_w^2/B_0^2, v_p/v_A) = (0.05, 0.07)$.

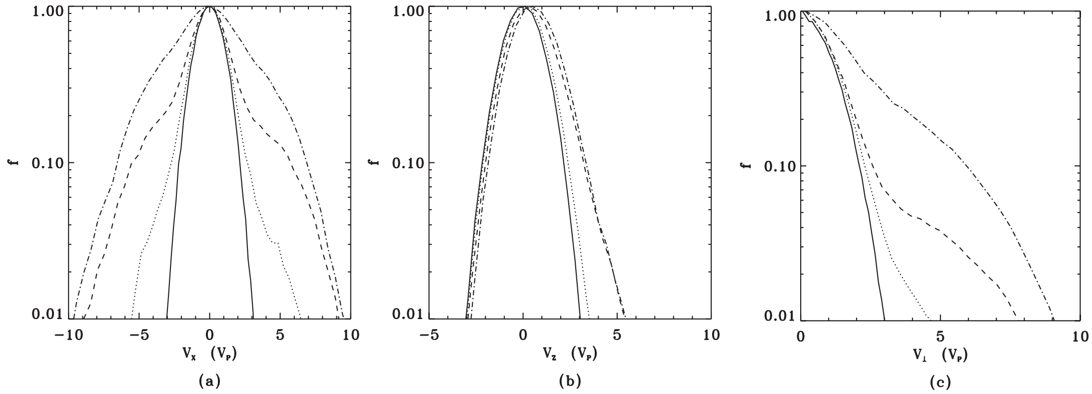


FIG. 2. The normalized velocity distribution functions plotted against $v_x$ (a), $v_z$ (b), and $v_\perp$ (c), for the same input parameters as in Fig. 1. The solid line, dots, dashed line, and dash-dots correspond to time intervals $\Omega t = 0$, 2, 4, and 7, respectively.

tion with thermal speed $v_p$, which is less than $v_A$ to ensure that the cyclotron resonance condition cannot be satisfied.

In Fig. 1, we present scatter plots in the $v_x$-$v_y$ and $v_\perp$-$v_z$ space, which shows the process of particle heating in a self-explanatory manner. Here, the velocity is normalized to the initial thermal speed $v_p$. The input parameters are $v_p = 0.07 v_A$ and $\delta B_w^2/B_0^2 = 0.05$. In Fig. 2 we show the distribution function for the same case. The solid line, dots, dashed line, and dash-dots correspond to time intervals, $\Omega t = 0, 2, 4, 7$, in that order. Figure 3 shows the temporal evolution of the kinetic temperatures, where the results based on four sets of input parameters $(\delta B_w^2/B_0^2, v_p/v_A) = (0.05, 0.07)$, $(0.12, 0.07)$, $(0.05, 0.35)$, and $(0.12, 0.35)$ are presented by the solid line, the dots, the dash-dots, and the dashed line, respectively. In Fig. 4, we present the kinetic temperature anisotropy at different initial thermal speeds. For these cases, the bulk velocity of the particles in the parallel direction is generally less than $0.1 v_A$, which is much lower than the square-root mean value calculated. The implication is that the protons are significantly heated.

The numerical results are qualitatively consistent with and in a reasonable agreement with the analytic results. For the sake of illustration, the analytic expression for the temperatures $T_z/T_i$ and $T_\perp/T_i$ are superposed in Fig. 3. Here, we should note that the predicted large anisotropy may not be observable in nature, as such a feature is expected to drive a number of anisotropy-driven instabilities [18,19], which in turn will quickly reduce the anisotropy. Among them is the ion cyclotron instability, whose threshold condition is $T_\perp/T_z \approx 2 \sim 3$.

In summary, we conclude that Alfvén waves can result in nonresonant pitch-angle scattering of ions in low-beta plasmas. This process randomizes the ion orbits and leads to higher kinetic temperatures in directions transverse and parallel to the ambient magnetic field. In this Letter we demonstrate, both analytically and numerically, the heating of the ions by a spectrum of Alfvén waves via such a scattering process. The present discussion enables us to better our understanding of the physics behind some of the simulation results [20,21]. In the article by Li *et al.* [20], whose main focus was on the pickup of energetic ions, it was found that enhanced Alfvén waves excited by the energetic ions led to the heating of the background ions, although the customary theory predicts that the background ions should be impervious to the excitation of Alfvénic turbulence. The present Letter, however, provides a clear physical explanation.

We stress that the wave-particle interaction process discussed in this Letter is not linear. The nonlinearity enters the problem implicitly through the wave magnetic and electric fields in the equation of motion (2). Although the wave **E** and **B** fields are not solved self-consistently, the problem is conceptually and intrinsically nonlinear in nature. In fact, the nonlinearity is clearly manifested by the fact that the heating is proportional to the square amplitude of the wave magnetic field. We also note that the particle energy is indeed conserved during pitch-angle scattering but it is in the wave frame, not the stationary frame of the

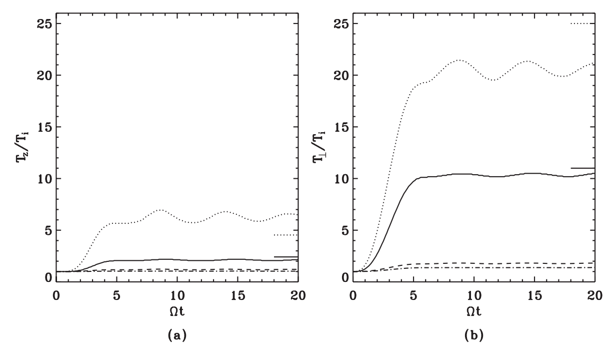


FIG. 3. The temporal evolution of the parallel (a) and perpendicular (b) kinetic temperatures normalized with respect to their initial values $T_i$. The various input parameters, $(\delta B_w^2/B_0^2, v_p/v_A) = (0.05, 0.07)$, $(0.12, 0.07)$, $(0.05, 0.35)$, and $(0.12, 0.35)$ are represented by the solid line, the dots, the dash-dots, and the dashed line, respectively. The analytic results based on Eq. (8) are indicated by the horizontal short lines for the cases $(\delta B_w^2/B_0^2, v_p/v_A) = (0.05, 0.07)$ and $(0.12, 0.07)$.

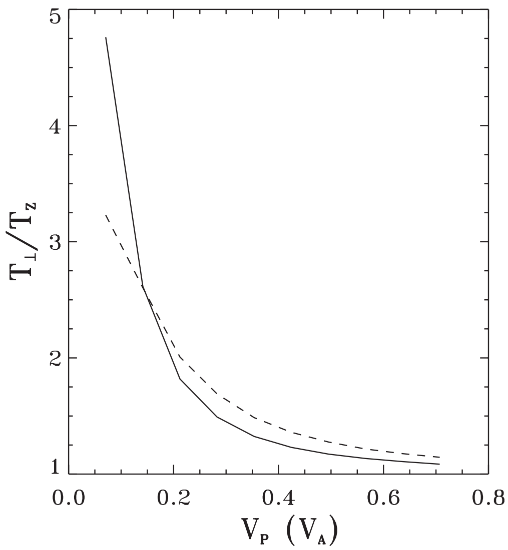


FIG. 4. The plot of kinetic temperature anisotropy vs initial thermal speed $v_p$, where $T_\perp/T_z$ are the numerical values at time $\Omega t = 20$. The solid and dashed lines correspond to $\delta B_w^2/B_0^2 =$ 0.05 and 0.12, respectively.

particles. Since in the laboratory frame the waves are propagating relative to the particles, it is quite clear that the particles should gain energy by being pitch-angle scattered by the Alfvén waves. Here we remark that the pitch-angle scattering process itself is an intrinsically nonlinear phenomenon. Linear resonance theory cannot describe such a process.

Finally, we should reiterate that the physical process considered in Chen *et al.* [15] is different from what is considered in this Letter. Because in their discussion they consider that the Alfvén wave is monochromatic, a finite wave vector $\mathbf{k}_\perp$ is necessary to ensure that the system has more than 1 degree of freedom, hence, the possible transition to chaos. On the other hand, in our discussion we consider a spectrum of Alfvén waves that randomize the velocity distribution of an ensemble of particles. Although initially this process depends upon the positions of the particles, subsequently these particles tend to mix and they would forget their initial positions. This process takes place even if the waves are propagating along the ambient magnetic field. However, the major conclusions obtained in Ref. [15] are qualitatively in agreement with ours. For example, both papers find that heating in the perpendicular direction is more effective, and second, a significant level of the Alfvén wave field is needed to attain the meaningful heating although in our theory there is no threshold value of the wave field. Finally, we reiterate that, in our theory, significant heating occurs only if the plasma beta is low.

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*Electronic address: cbwang@ustc.edu.cn
†Electronic address: cswu@ustc.edu.cn
‡Electronic address: phy20723@yahoo.com

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