Commun. Theor. Phys. (Beijing, China) 34 (2000) pp 317–320
© International Academic Publishers
Vol. 34, No. 2, September 15, 2000

Numerical Studies on the Formation of Magnetic Loops*

WANG ChuanBing, ZHENG HuiNan and WANG Shui

Department of Earth and Space Sciences, University of Science and Technology of China, Hefei 230026, China

(Received July 22, 1998; Revised July 1, 1999)

Abstract *The formation of a current-carrying non-force-free magnetic loop is studied by three-dimensional magnetohydrodynamic simulation. The effects of the plasma pressure and the gravitational force are considered for the first time. The results show that the twist motion of the footpoints of magnetic field lines at the photosphere will form magnetic loop in the corona. The magnetic loop expands mainly in both the vertical and horizontal directions along the magnetic neutral line at the photosphere bases.*

PACS numbers: 52.65.-y, 96.60.Hv, 96.60.Pb
Key words: magnetic loops, formation and evolution

Observed clearly by the soft X-ray telescope of Skylab and Yohkoh spacecrafts,[1] corona loops are believed to play a crucial role in a large variety of phenomena such as flares, corona mass ejections, prominences and corona heating.[2] These soft X-ray-emitting loop structures are considered generally identifying the paths of magnetic field lines, so they are magnetic loops. On the theoretical studies of magnetic loops during the past two decades, most published papers have been addressed the problem in the framework of a simple model in which the field occupies the half-space and is y-invariant,[3,4] or in antisymmetric cylindrical geometry in which the problem can be reduced to two dimensions.[5] Only very recently, several authors investigated the fully three-dimensional problem. However, linear or nonlinear force-free magnetic fields are often introduced or obtained in these three-dimensional simulation works.[6–8] The non-force-free magnetic field may be taken by some authors, but they just studied in simple cylindrical geometry.[9] In this paper, we will present a three-dimensional magnetohydrodynamic (MHD) simulation to investigate the formation of current-carrying non-force-free magnetic loops. We include the influences of plasma pressure and gravitational force for the first time, which are important in phenomena such as prominence.[10]

The basic equations to be solved are the following compressible, nonresistive, viscous MHD equations, which are written in nondimensional form,

$$\frac{\partial \rho}{\partial t} = -\boldsymbol{V}\cdot\nabla\rho - \rho\nabla\cdot\boldsymbol{V}\,, \qquad \frac{\partial \boldsymbol{V}}{\partial t} = -\boldsymbol{V}\cdot\nabla\boldsymbol{V} - \frac{\beta_0}{2\rho}\nabla P - \frac{1}{\rho}(\nabla\times\boldsymbol{B})\times\boldsymbol{B} + \frac{v}{\rho}\nabla^2\boldsymbol{V} - \boldsymbol{g}\,,$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla\times(\boldsymbol{V}\times\boldsymbol{B})\,, \qquad \frac{\partial P}{\partial t} = -\boldsymbol{V}\cdot\nabla P - \gamma P\nabla\cdot\boldsymbol{V}\,. \tag{1}$$

where ρ, $\boldsymbol{V}$, $\boldsymbol{B}$, P and $\boldsymbol{g}$ are the mass density, flow velocity, magnetic field, pressure and normalized gravitational acceleration that directed along the negative z-axis, respectively. v is the coefficient of viscosity, which is uniform and constant $v = 10^{-4}$ in this paper. γ is the special heat ratio $\gamma = 5/3$. β_0 is the ratio of the plasma pressure to magnetic field pressure, which is set as $\beta_0 = 0.2$ in this simulation. The normalized characteristic quantities are the magnetic field $\boldsymbol{B}_0$, mass density ρ_0, plasma pressure P_0, length L_0, Alfvén speed $V_A = B_0/\sqrt{\mu_0\rho_0}$ and Alfvén time $\tau_A = L_0/V_A$.

We use a three-dimensional Cartesian coordination system. The simulation domain is given by a rectangular box $\{-L_x \le x \le L_x,\ -L_y \le y \le L_y,\ 0 \le z \le L_z\}$, the size of it is large enough compared with the characteristic spatial scale of the system. In this paper, we take $L_x = L_y = 10$ and $L_z = 20$.

*The project supported by National Natural Science Foundation of China (No. 19573009) and Ministry of Science and Technology of China

The initial magnetic field is a bipolar potential magnetic field embodying in an isothermal gravitational stratified plasma

$$[B_x, B_y, B_z] = 4[x^2 + y^2 + (z+1.5)^2]^{-5/2}\,[2x^2 - y^2 - (z+1.5)^2, 3xy, 3x(z+1.5)] \tag{2}$$

and the normalized initial mass density and pressure are given by

$$\rho = P = \exp\left(-2gz/\beta_0\right). \tag{3}$$

Two cases are studied in our simulation. In case 1, the system develops freely without introducing any plasma motion at the photosphere boundary. In case 2, for $t \geqslant 0$, we impose a plasma convection by which the footpoints of the bipolar magnetic field on the bottom of the simulation box are driven to twist at the photosphere base. The convection motion imposed at the base is derived by the stream function $\phi[B_z(x, y, 0)]$, which is defined by

$$\phi(B_z) = v_0 B_z^2 \exp\left[\frac{B_z^2 - B_{z\max}^2}{\delta B^2}\right], \tag{4}$$

where $B_{z\max}$ is the maximum value of $B_z(x, y, 0)$, $\delta B = 10$ and $v_0 = 0.05$. The flow velocity at the base is then given by $V_x = -(\partial\phi/\partial y)\,f(t)$, $V_y = (\partial\phi/\partial x)\,f(t)$, where the function $f(t)$ is a smooth increasing function to avoid a numerical artifact due to the sudden start of the rotation at $t = 0$, which is $0.5\,[1 - \cos(\pi t/T_s)]$ when $t < T_s$, and is 1 when $t \geq T_s$. In this paper, we take $T_s = 4$.

Equations (1) can be numerically solved by the fractal step method, which enables us to solve problems that could not be treated by means of ordinary difference scheme-scheme of simple approximation, where conditions of stability and consistency must be satisfied at each step. Inside the simulation box, the MHD equations are discretized on a nonuniform mesh with $81 \times 81 \times 81$ grids. The implicit second-order central difference is used for the diffusion term, while the other spatial derivative is approximated by the implicit upwind scheme. The boundary conditions used in our simulation are the following. At the bottom boundary of box, the normal component B_z of magnetic field is preserved by the convection velocity field, meanwhile the horizontal components are calculated directly from Eq. (1) by using forward of backward difference. On the other faces of box, we take the nonreflecting boundary conditions. In order to assure the numerical accuracy of the solenoidal condition for the magnetic field, the divergence-cleaning procedure is implemented in the present calculation.[11]

The time development of the magnetic field lines for the bipolar magnetic loop is shown in Fig. 1 for case 2. Plots in this figure are 10 representative field lines which start from the points located on the positive spot center (black) and the negative spot center (gray), where the normal magnetic field B_z has the maximum value. As one can see in Fig. 1, the magnetic field lines are being twisted more and more strongly due to the convection motion at their footpoints. At the same time, the magnetic loop expands both in the vertical direction and in the horizontal direction along the magnetic neutral line at the photosphere base, but it nearly keeps the same size as its initial status along the horizontal direction perpendicular to the magnetic neutral line. This is different from the results obtained by Ozaki and Sato,[8] their results showed the bipolar flux loop expanded mainly in the vertical and horizontal directions perpendicular to the magnetic neutral line in their incompressible MHD simulation. The geometry of magnetic field line nearly keeps the same geometry as its initial status in case 1.

More quantitative information on the evolution of the magnetic loop is shown in Fig. 2 by presenting the time variation of system energy and length of a selected magnetic field line. This selected magnetic field line emerges from the point of bipolar spot center where the normal magnetic field B_z is maximum. It can be seen that the length of magnetic field line is a monotonically increasing function of time. The magnetic flux tube expands to about 6 times as large as the initial one when $t = 120\tau_A$. All of energies are normalized to the magnetic energy $B_0^2/2\mu_0$. As can be seen, the relative changes of the magnetic energy and flow kinetic energy increase monotonically with time, which appear to approach an upper limit. The relative changes of gravitational potential energy and thermal energy mainly decrease to a negative value with time after an initially brief increase. This may be due to more plasma moving out of the simulation region in case 2 than in case 1. Moreover, a relatively steady magnetic loop is obtained when $t = 120\tau_A$, because the magnetic energy, flow energy of the system and length of selected field line are all changing with time only slightly. The magnetic energy of magnetic loop is finally about 1.45 times as large as that of initial potential magnetic field.

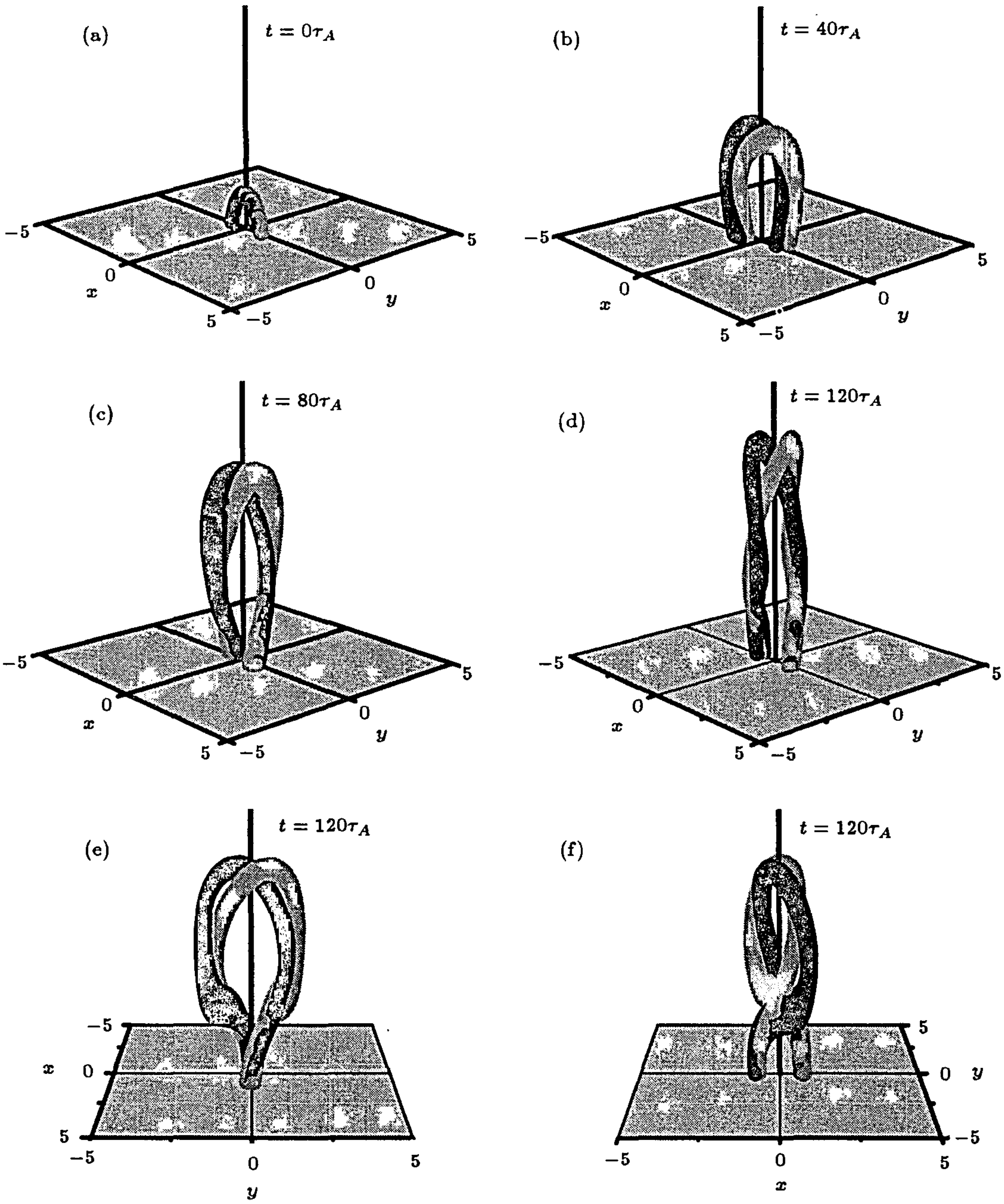

Fig. 1. Idea MHD evolution of the magnetic structure due to photosphere convection, which is found to form a relative steady magnetic loop at about $t = 120\tau_A$. The figures show the evolution of a set of selected field lines started from the spot center at four particular stages corresponding to (a) $t = 0\tau_A$; (b) $t = 40\tau_A$; (c) $t = 80\tau_A$; and (d) $\sim$ (f) $t = 120\tau_A$.

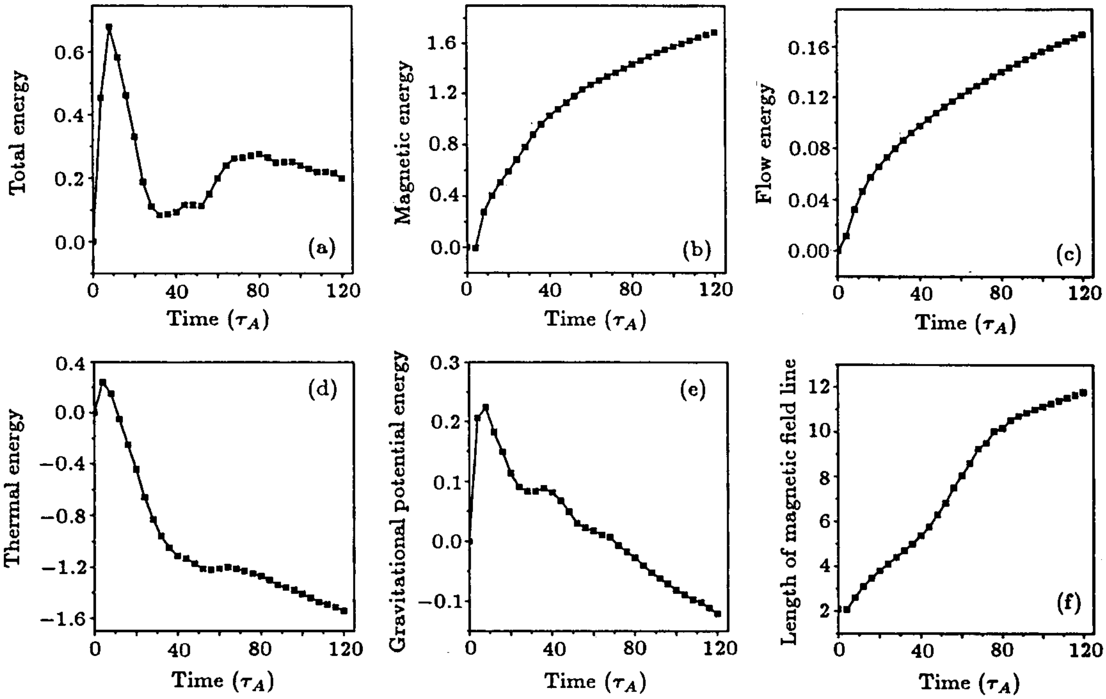

Fig. 2. Variation of various types of energy of the system and the length of a selected magnetic field line with time. Subtractions of energies, namely, $\Delta E = E_2 - E_1$, are shown in (a) ~ (e), where E_1 and E_2 are the corresponding energies in case 1 and case 2 respectively. (f) Length of a magnetic field line in case 2.

In this paper, the formation of magnetic loops has been numerically studied by fully three-dimensional MHD simulation. We obtained the final magnetic loop through twisting the footpoints of a potential bipolar magnetic field. The effects of the plasma pressure and gravitational force are included for the first time in our simulation, so the final magnetic loop is a current-carrying non-force-free magnetic field. The simulation results show that the magnetic loop expands mainly along the vertical and horizontal directions parallel to the magnetic neutral line at the photosphere base, whose size can be 6 times as large as the one of the potential magnetic flux tube. The stored magnetic energy in the magnetic loop is finally about 1.45 times larger than its initial one.

References

[1] R. Kano and S. Tsuneta, Astrophys. J. **454** (1995) 934.

[2] E.R. Priest, *Physics of Magnetic Flux Ropes*, eds C.T. Russel *et al.*, American Geophysical Union, Washington DC (1990) p. 1.

[3] M. Ugai, Phys. Plasmas **3** (1996) 4172.

[4] K. Shibata, T. Tajima, *et al.*, Astrophys. J. **345** (1989) 584.

[5] S. Poedts and G.C. Boynton, Astron. Astrophys. **306** (1996) 610.

[6] J.A. Klimchuk and P.A. Sturrock, Astrophys. J. **385** (1992) 344.

[7] T. Amari, J.F. Luciani, *et al.*, Astrophys. J. **466** (1996) L39.

[8] M. Ozaki and T. Sato, Astrophys. J. **481** (1997) 524.

[9] J.I. Sakai and C. de Jager, Solar Phys. **173** (1997) 347.

[10] B.C. Low, Astrophys. J. **399** (1992) 300.

[11] J.D. Ramshaw, J. Compt. Phys. **52** (1983) 592.