Pitch-angle diffusion of ions via nonresonant interaction with Alfvénic turbulence

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The present discussion revisits the problem of nonresonant heating of ions by Alfvénic turbulence. It is shown that in the limit of weak Alfvénic turbulence it is appropriate to describe the nonresonant heating of protons as perpendicular pseudoheating. However, in a more general situation it is demonstrated that the more appropriate view of the nonresonant heating process is the pitch-angle scattering in the wave frame. The purpose of this paper is to generalize the earlier theory to the case in which the energy density of the turbulent Alfvén waves is not necessarily very low. For weakly turbulent situation the present analysis confirms the earlier finding by Wu and Yoon [Phys. Rev. Lett. **99**, 075001 (2007)], according to whom the nonresonant Alfvén wave heating is described as leading to perpendicular pseudoheating of the protons. However, for more general situation the present paper demonstrates that pitch-angle scattering plays the principal role in the Alfvén wave pseudoheating process, and thereby shows that the perpendicular heating discussed by Wu and Yoon is kinetic in nature, not attributable to fluid motion. © *2009 American Institute of Physics*.
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I. INTRODUCTION

A theory of nonresonant heating of protons by Alfvén waves is presented in Ref. 1 according to which, if T_0 is the initial proton temperature, then the nonresonant interaction with Alfvén waves leads to an "apparent" increase in the perpendicular temperature as given by

$$T_{\perp} = T_0 + \frac{W}{n_p}, \tag{1}$$

where $W = B_W^2/8\pi = (8\pi)^{-1}\int d\mathbf{k}|B_{\mathbf{k}}|^2$ is the wave magnetic field energy density and n_p is the thermal proton density. According to magnetohydrodynamics theory coherent Alfvén waves with magnetic field $\mathbf{B}_W$ may induce fluid motion with velocity $\mathbf{v}_0$ (Ref. 2)

$$\mathbf{v}_0 = \frac{\mathbf{B}_W}{\sqrt{4\pi n_p m_p}}. \tag{2}$$

The induced fluid kinetic energy density is therefore

$$\frac{1}{2}m_p v_0^2 = \frac{B_W^2}{8\pi n_p} = \frac{W}{n_p}. \tag{3}$$

Here we should note that Ref. 1 is based on kinetic theory of turbulent Alfvén waves. Even though Eqs. (1) and (3) share some resemblances, their physical natures are fundamentally different. This point is expounded in more detail in our recent paper.[3]

The nonresonant Alfvén wave heating theory proposed in Ref. 1 is important in the context of the solar coronal heating and solar wind acceleration problem. In the literature, one of the widely accepted views is that coronal heating and solar wind acceleration is intimately related to Alfvén waves.[4–9] However, most discussions focus on resonant cyclotron wave-particle interaction. Against this backdrop Ref. 1 suggests that nonresonant interactions may lead to a substantial increase in proton thermal energy.

The essential findings in Ref. 1 are first, the nonresonant heating process does not involve dissipation of wave energy. Heating in the customary sense requires some form of dissipation, however. Second, the randomized proton motion implicit in the discussion is actually parasitic to turbulent waves. As such, it is implied that if the wave energy density should subside, then the original particle motion without the wave field should be restored. As a matter of fact, Ref. 10 carried out just such a numerical experiment in which it was shown that the proton temperature returned to its original value when the waves were allowed to subside. Consequently, the apparent heating discussed in Refs. 1, 3, and 10 is "pseudoheating."

The purpose of this paper is to generalize the earlier theory to the case in which the energy density of the turbulent Alfvén waves is not necessarily very low. For weakly turbulent situation, the analysis in Ref. 1 is applicable, according to which, the nonresonant Alfvén wave heating is described as leading to perpendicular pseudoheating of the protons. The present analysis demonstrates that pitch-angle scattering plays the principal role in the Alfvén wave pseudoheating process and thereby confirms once more that the perpendicular heating discussed in Ref. 1 is kinetic in

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nature, not attributable to fluid motion, despite the superficial similarity associated with Eqs. (1) and (3).

The organization of the present paper is as follows: in Sec. II we first present an alternative approach to quasilinear theory that includes nonresonant wave-particle interaction. Subsequently we apply the theory to thermal protons in Sec. III. Finally we present discussion and conclusions in Sec. IV.

II. QUASILINEAR THEORY INVOLVING NONRESONANT INTERACTIONS

The quasilinear kinetic equation for electromagnetic waves is reviewed in several publications—see, e.g., Refs. 11 and 12. For Alfvén waves or magnetosonic waves, the kinetic equation is greatly simplified because $k_\perp v_\perp/\Omega_s \ll 1$. On the basis of the discussion presented in the Appendix we generalize the desired kinetic equation to include nonresonant wave-particle interactions,

$$\frac{\partial F_s}{\partial t} = \frac{e_s^2}{4m_s^2}\sum_{n=\pm 1}\int d\mathbf{k}\frac{1}{v_\perp}\hat{R}\times\left\{v_\perp\left[\pi\delta(\omega - n\Omega_s - k_z v_z)\times|E_\mathbf{k}|^2 - \frac{\partial}{2\,\partial\omega}\left(\mathcal{P}\frac{1}{\omega - n\Omega_s - k_z v_z}\right)\frac{\partial|E_\mathbf{k}|^2}{\partial t}\right]\hat{R}F_s\right\}, \tag{4}$$

where the subscript s indicates ion species; $|E_\mathbf{k}|^2$ represents the wave electric field associated with Alfvén or magnetosonic waves; $\Omega_s = e_s B_0/m_s c$ is the gyrofrequency; e_s and m_s are charge and mass, respectively; and B_0 is the ambient magnetic field intensity, and the operator $\hat{R}$ is defined by

$$\hat{R} = \left(1 - \frac{k_z v_z}{\omega}\right)\frac{\partial}{\partial v_\perp} + \frac{k_z v_\perp}{\omega}\frac{\partial}{\partial v_z}. \tag{5}$$

For Alfvén waves resonant wave-particle interactions cannot occur for thermal protons since $\Omega_s \gg \omega$ and $\Omega_s \gg k_z v_z$. Consequently, we have

$$\frac{\partial}{2\,\partial\omega}\left(\mathcal{P}\frac{1}{\omega - n\Omega_s - k_z v_z}\right)\frac{\partial|E_\mathbf{k}|^2}{\partial t} \approx -\frac{1}{2\Omega_s^2}\frac{\partial|E_\mathbf{k}|^2}{\partial t}, \tag{6}$$

so that Eq. (4) reduces to

$$\frac{\partial F_s}{\partial t} = \frac{1}{8\pi m_p n_p}\int d\mathbf{k}\frac{1}{v_\perp}\frac{\partial|B_\mathbf{k}|^2}{\partial t}\hat{R}(v_\perp \hat{R}F_s), \tag{7}$$

where $B_\mathbf{k}$ is the magnetic field associated with Alfvén waves; n_p and m_p denote proton number density and proton mass, respectively.

Reference 1 derives the same equation by replacing the delta function with the following nonresonant approximation:

$$\frac{\gamma}{(\omega \pm \Omega_s - k_z v_z)^2 + \gamma^2} \approx \frac{\gamma}{\Omega_s^2} \rightarrow \frac{1}{2\Omega_s^2}\frac{\partial}{\partial t}, \tag{8}$$

which is a customary practice in the literature.[11] Even though this procedure leads to formally identical result with Eq. (6), the customary approach leads to the issue of self-consistent determination of γ. In contrast, the advantage of Eq. (6) is that one does not need to invoke finite γ at all, so that the theory becomes applicable to a situation where the plasma is stable ($\gamma=0$) but the turbulent Alfvén waves are generated elsewhere, hence $\partial|E_\mathbf{k}|^2/\partial t$ varies with time simply due to external wave source.

The kinetic energy of each ion is conserved in the Alfvén wave frame. Consequently it is convenient to work with a spherical coordinate system defined in the wave frame to describe the distribution function. In such a representation, Eq. (7) reduces to pitch-angle diffusion equation,

$$\frac{\partial F_s(v,\mu,\tau)}{\partial\tau} = \frac{\partial}{\partial\mu}\left((1-\mu^2)\frac{\partial F_s(v,\mu,\tau)}{\partial\mu}\right), \tag{9}$$

where $\mu = \cos\theta$ and θ is the pitch angle defined in the wave frame. Moreover, τ is the normalized "time" variable defined by

$$\tau \equiv \frac{B_W^2}{2B_0^2} = \int d\mathbf{k}\frac{B_k^2}{4B_0^2}. \tag{10}$$

Note that the variable τ represents the turbulence level rather than an actual time. If $\tau \ll 1$, then it represents weak turbulence. Equation (9) and its solution can be applied to moderate values of τ. Consequently, the present formalism is capable of dealing turbulence that is weak but not necessarily very weak.

Let us assume a functional form for the velocity distribution function,

$$F_s(v,\mu,\tau) = G_s(v)f_s(\mu,\tau). \tag{11}$$

In the low-beta limit we may consider a particularly simple form

$$G_s(v) = \frac{\delta(v - v_s)}{2\pi v_s^2}, \tag{12}$$

where v_s denotes the initial velocity (at $\tau=0$) of the ion species s defined in the wave frame. With the above consideration the reduced distribution $f_s(\mu,\tau)$ satisfies the same pitch-angle diffusion Eq. (9). The general solution is given by

$$f_s(\mu,\tau) = \sum_{l=0}^{\infty}\left(l + \frac{1}{2}\right)P_l(\mu)\times\int_{-1}^{1} d\bar{\mu}P_l(\bar{\mu})e^{-l(l+1)\tau}f_s(\bar{\mu},0), \tag{13}$$

where $P_l(\mu)$ is the Legendre polynomial of order l.

For a simple initial distribution given by

$$f_s(\mu,0) = \delta(\mu - \mu_0), \tag{14}$$

where μ_0 is the initial value of μ at $\tau=0$, the solution (13) reduces to have

$$f_s(\mu,\tau) = \sum_{l=0}^{\infty}\left(l + \frac{1}{2}\right)P_l(\mu)P_l(\mu_0)e^{-l(l+1)\tau}. \tag{15}$$

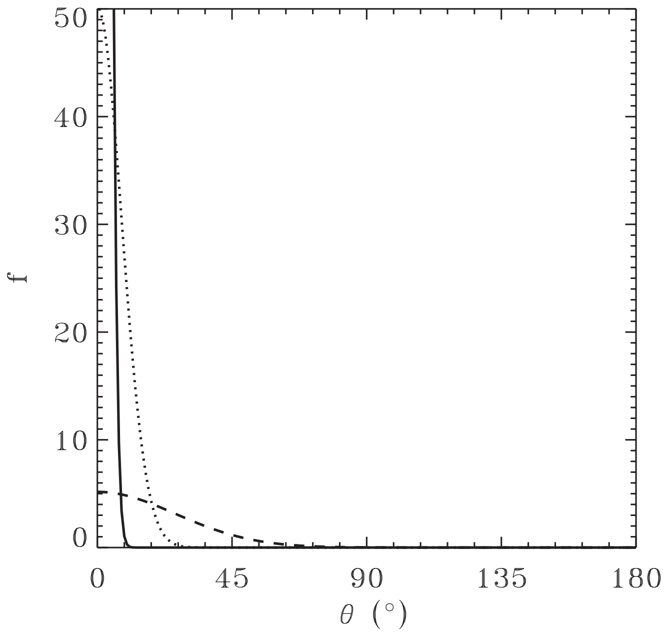

FIG. 1. Three different wave energy levels, namely, $\tau=0.001$ (solid line), 0.01 (dots), and 0.1 (dashes), are considered. Pitch-angle diffusion is enhanced for increasing τ. For weak turbulence (small τ), the pitch angle distribution diffusion affects mainly over a narrow region near $\theta \approx \theta_0 = 0$.

III. THERMAL PROTONS

Let us pay attention mainly to thermal protons ($s=p$) henceforth. Let us assume that the wave-frame velocity distribution is given by

$$G_p(v) = \delta(v - v_A)/(2\pi v_A^2), \tag{16}$$

and that $\mu_0 = 1$. In the limit of small τ (i.e., weak turbulence), solution (15) may be approximately written as[13]

$$f_p(\theta,\tau) \approx \frac{1}{2\tau}\exp\left(-\frac{\theta^2}{4\tau}\right), \tag{17}$$

for small pitch angle θ. If we write $v_A\theta \approx v_\perp$ and make use of the relation

$$4v_A^2\tau = \frac{B_W^2}{4\pi n_0 m_p} = \frac{2T_\perp}{m_p}, \tag{18}$$

where

$$T_\perp \equiv \int dk \frac{B_k^2}{8\pi n_0} = \frac{B_W^2}{8\pi n_0}, \tag{19}$$

is the "apparent perpendicular temperature" discussed in Ref. 1, then we obtain

$$f_p = \frac{m_p}{2T_\perp}\exp\left(-\frac{m_p v_\perp^2}{2T_\perp}\right), \tag{20}$$

which is comparable to that obtained in Ref. 1, provided we further assume that the initial temperature is zero. The above discussion shows that in the limit of weak Alfvénic turbulence it is appropriate to describe the proton distribution function in terms of the coordinates $(v_\perp, v_z)$.

Figure 1 plots solution (15). In Fig. 1 we consider three different wave energy levels, $\tau=0.001$ (solid line),

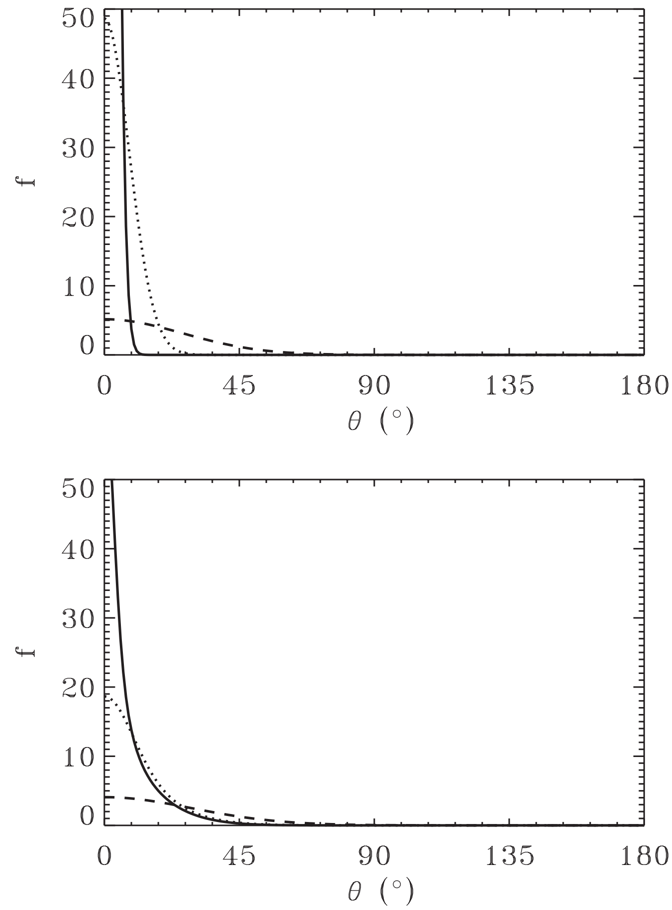

FIG. 2. Solution based upon the initial condition Eq. (21). We consider $\tau=0.001$ (solid line), 0.01 (dots), and 0.1 (dashes), and two case of Δ^2 are considered, namely, $\Delta^2=0.025$ (top) and $\Delta^2=0.25$ (bottom).

0.01 (dots), and 0.1 (dashes). Pitch-angle diffusion is seen to be enhanced as the wave energy density increases, that is, for increasing τ. For weak turbulence (small τ), the pitch angle distribution broadens mainly over a narrow region near $\mu \approx \mu_0 = 1$. For such a situation it is appropriate to describe the evolution of proton distribution function as heating along $v_\perp$, as noted above. However, for higher τ (turbulence that is not necessary *very* weak) the evolution of proton distribution is more accurately described as diffusion in pitch-angle space rather than perpendicular heating.

We may also generalize the initial distribution to have a finite velocity spread. For instance, we may take

$$f_p(\mu,0) = N\exp\left(-\frac{\theta^2}{\Delta^2}\right), \tag{21}$$

where N is a normalization constant $N^{-1} = \int_{-1}^{1} d\mu \times \exp(-\theta^2/\Delta^2)$. Figure 2 displays the solution based upon the initial condition given by Eq. (21). We consider $\tau=0.001$ (solid line), 0.01 (dots), and 0.1 (dashes), and two cases of the initial pitch-angle spread are considered, namely, $\Delta^2 = 0.025$ and $\Delta^2=0.25$.

To recap the discussion thus far, we have revisited the problem of nonresonant Alfvén wave interaction from the perspective of pitch-angle diffusion in the present paper. In contrast, Ref. 1 approached the same problem from the standpoint of (pseudo)heating. However, it should be pointed out that diffusion and heating are basically one and the same. As we show in the present discussion, for weak turbulence pitch-angle diffusion can be alternatively interpreted as perpendicular ion heating.

IV. DISCUSSION AND CONCLUSION

To conclude the present discussion, we showed that in the limit of weak Alfvénic turbulence it is appropriate to describe the nonresonant heating as perpendicular pseudoheating. However, in general it is more appropriate to view the nonresonant heating process as pitch-angle diffusion (or scattering) in the wave frame. Here we reiterate that the nonresonant heating discussed in Ref. 1 and in the present discussion is a "pseudo" heating process, which is conceptually different from the case in which coherent Alfvénic wave is present[2]—for further discussion of this issue see the test particle calculation carried out in Refs. 10 and 14.

In a recent study of minor ion heating by Alfvén waves via nonresonant interactions, Bourouaine *et al.*[15] employ the kinetic equation derived in Ref. 1, but without imposing the low-beta approximation. They also discuss the effects of collisions on the quasilinear process. As expected, it is found that collisions tend to isotropize the heating.

In the context of the present analysis we investigate the effects of initial pitch-angle on the diffusion process. Such a study may be of interest for minor ions. We thus show in Figs. 3–5, the solution with arbitrary initial pitch angle θ_0. We consider two types of initial distribution. The first case is

$$f_p(\mu,0) = \delta(\theta - \theta_0), \tag{22}$$

while in the second case we consider

$$f_p(\mu,0) = N \exp\left(-\frac{(\theta-\theta_0)^2}{\Delta^2}\right), \tag{23}$$

where $N^{-1}=\int_{-1}^{1} d\mu \exp[-(\theta-\theta_0)^2/\Delta^2]$. For all the cases we consider several values of the initial pitch angles $\theta_0=30°$, 60°, 90°, and 135°. We also choose $\tau=0.001$ (solid lines), 0.01 (dots), and 0.1 (dashes) for each case of θ_0.

In Fig. 3, we consider the delta-function initial distribution (22). The identification of initial pitch angles, θ_0 is self-explanatory, but for the sake of completeness, the cases of $\theta_0=30°$, 60°, 90°, and 135° are from top-down in that order. Also, for each θ_0, $\tau=0.001$ (solid lines), 0.01 (dots), and 0.1 (dashes) are considered.

We next consider the initial distribution with thermal spread (23). Figure 4 considers $\Delta^2=0.025$ and Fig. 5 considers $\Delta^2=0.25$. It is clear that the larger the initial pitch angle the more effective the diffusion process, and higher the initial spread of pitch-angles more diffused is the final state of the distribution function in pitch-angle space.

Finally we remark that the physics of pitch-angle scattering of ions by Alfvén waves were extensively discussed in late 1990s.[16] The study was stimulated by commetary and solar wind research. However, most of the discussions were concerned with energetic ions streaming with velocities much higher than the Alfvén speed. These ions can interact with Alfvén waves via cyclotron resonance. The scattering of thermal protons by nonresonant wave particle interactions was not conceived until very recently. In this regard, the present discussion may be significant and useful for other researchers.

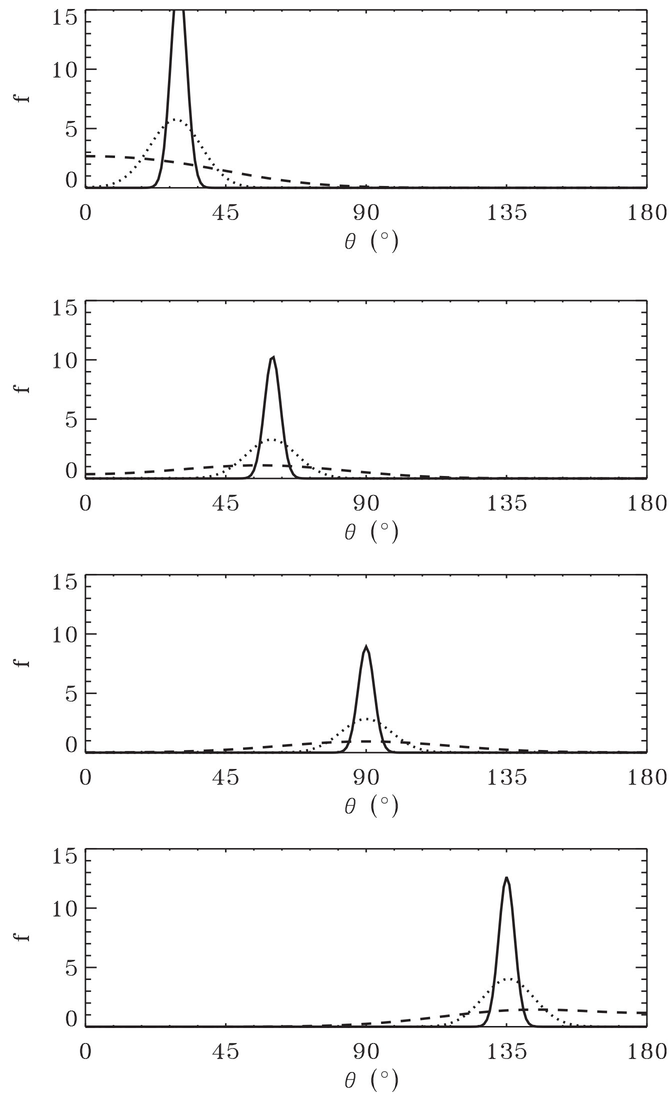

FIG. 3. Solution based upon the delta-function initial distribution (22). Several values of the initial pitch angles, $\theta_0=30°$, 60°, 90°, and 135°, are considered, the identification of which is self-explanatory. We also choose $\tau=0.001$ (solid lines), 0.01 (dots), and 0.1 (dashes) for each case of θ_0.

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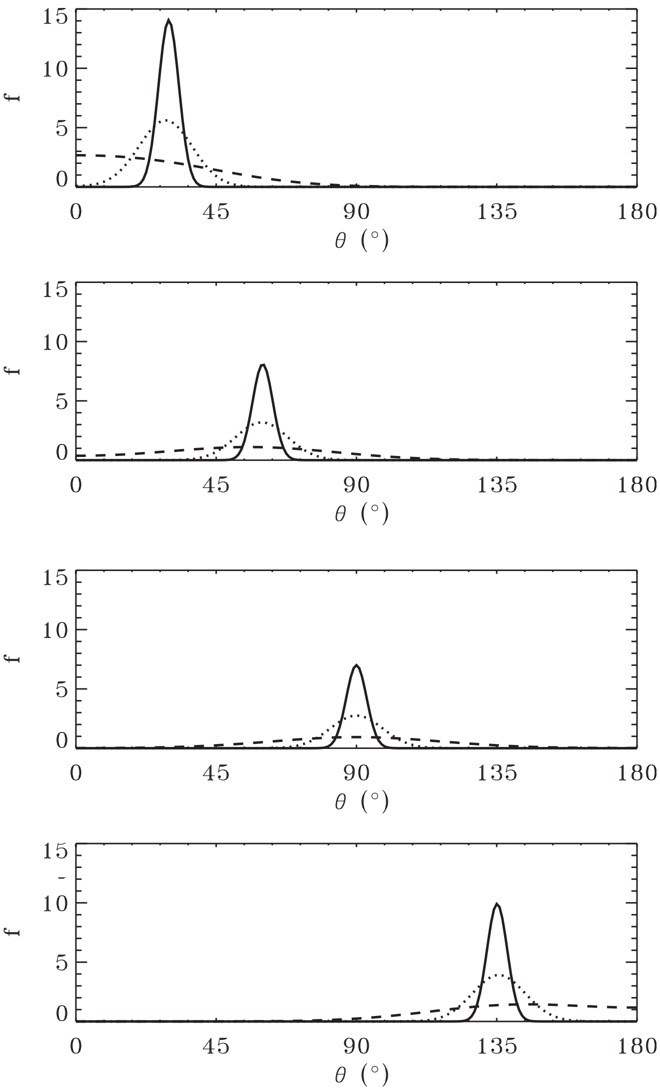

FIG. 4. Solution based upon the initial distribution with thermal spread (23) with $\Delta^2=0.025$. Several values of the initial pitch angles, $\theta_0=30°$, 60°, 90°, and 135°, are considered, the identification of which is self-explanatory. We also choose $\tau=0.001$ (solid lines), 0.01 (dots), and 0.1 (dashes) for each case of θ_0.

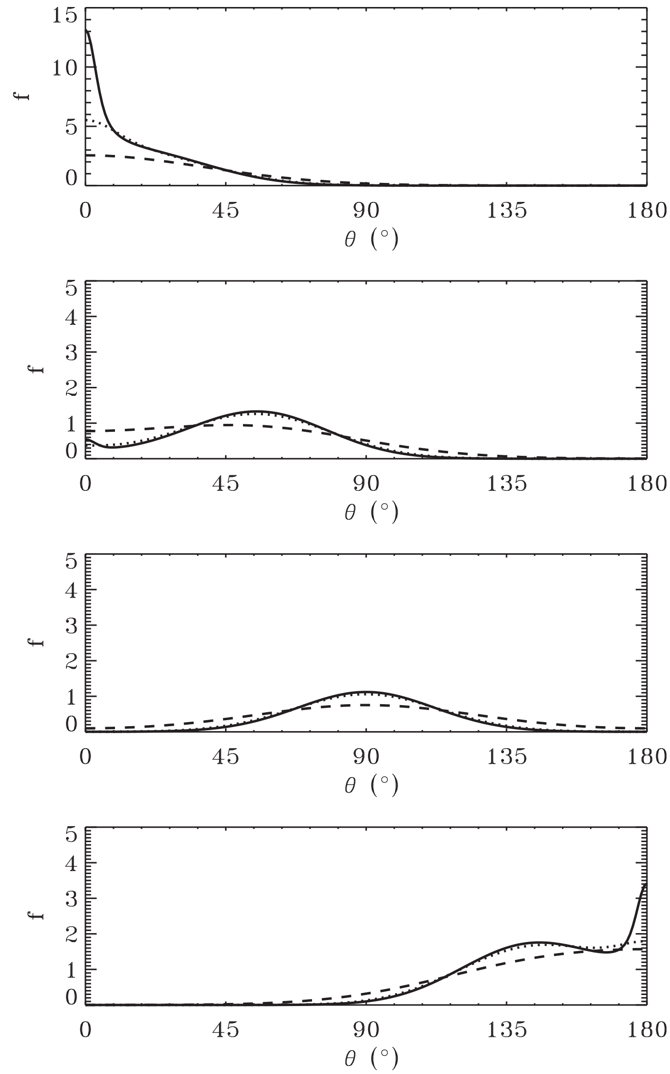

FIG. 5. Solution based upon the initial distribution with thermal spread (23) with $\Delta^2=0.25$. Several values of the initial pitch angles, $\theta_0=30°$, 60°, 90°, and 135°, are considered, the identification of which is self-explanatory. We also choose $\tau=0.001$ (solid lines), 0.01 (dots), and 0.1 (dashes) for each case of θ_0.

APPENDIX: QUASILINEAR THEORY INCLUDING NONRESONANT INTERACTIONS

In order to demonstrate the concepts associated with the present reformulation of the nonresonant quasilinear process, it is sufficient to consider the case of electrostatic waves in unmagnetized plasmas. We begin with the following linearized Vlasov equation

$$\frac{\partial f_s}{\partial t}+\mathbf{v}\cdot\nabla f_s+\frac{e_s}{m_s}\mathbf{E}\cdot\frac{\partial F_s}{\partial \mathbf{v}}=0,$$
$$\nabla\cdot\mathbf{E}=4\pi\sum_s e_s n_s\int d\mathbf{v} f_s(\mathbf{r},\mathbf{v},t), \tag{A1}$$

where F_s and f_s denote the average and perturbed distribution functions, respectively; and the subscript s denotes particle species. We consider that the wave electric field has two characteristic time scales t and εt where ε is a small parameter: the time dependence on t describes fast process such as the wave oscillation, while the time dependence of physical quantities on εt depicts slow temporal variation. The slow time dependence is compatible with quasilinear process. We first introduce a Fourier transform

$$\mathbf{E}(\mathbf{r},t,\varepsilon t)=\frac{1}{(2\pi)^3}\int d\mathbf{k}\hat{\mathbf{E}}(\mathbf{k},t,\varepsilon t)\exp(i\mathbf{k}\cdot\mathbf{r}),$$
$$f_s(\mathbf{r},\mathbf{v},t,\varepsilon t)=\frac{1}{(2\pi)^3}\int d\mathbf{k}\hat{f}_s(\mathbf{k},\mathbf{v},t,\varepsilon t)\exp(i\mathbf{k}\cdot\mathbf{r}). \tag{A2}$$

We then assume that the temporal dependence associated with the fast time scale is given by

$$\hat{\mathbf{E}}(\mathbf{k},t,\varepsilon t)=\hat{\mathbf{E}}(\mathbf{k},\varepsilon t)\exp(-i\omega_{\mathbf{k}}t),$$

$$\hat{f}(\mathbf{k},\mathbf{v},t,\varepsilon t)=\hat{f}(\mathbf{k},\mathbf{v},\varepsilon t)\exp(-i\omega_{\mathbf{k}}t),$$

where $\omega_{\mathbf{k}}$ denotes the wave frequency that satisfies an appropriate dispersion relation. It is supposed that the waves are in a quasistationary state and slow time rate of $\partial|\hat{E}(\mathbf{k},\varepsilon t)|^2/\partial t$ is known. We assume that in general both the wave field amplitude and the unperturbed distribution function vary on the slow time scale εt. Then the linearized kinetic equation may be written as

$$\frac{\partial\hat{f}_s(\mathbf{k},\mathbf{v},\varepsilon t)e^{-i\omega_{\mathbf{k}}t}}{\partial t}+i\mathbf{k}\cdot\mathbf{v}\hat{f}_s(\mathbf{k},\mathbf{v},\varepsilon t)e^{-i\omega_{\mathbf{k}}t}+\frac{e_s}{m_s}\hat{\mathbf{E}}(\mathbf{k},\varepsilon t)e^{-i\omega_{\mathbf{k}}t}\cdot\frac{\partial F_s(\mathbf{v},\varepsilon t)}{\partial\mathbf{v}}=0. \tag{A3}$$

Formal solution is given by

$$\hat{f}_s(\mathbf{k},\mathbf{v},\varepsilon t)=-\frac{e_s}{m_s}\int_{-\infty}^{t}dt'\,\hat{\mathbf{E}}(\mathbf{k},\varepsilon t')\times e^{-i(\omega_{\mathbf{k}}-\mathbf{k}\cdot\mathbf{v})(t'-t)}\cdot\frac{\partial F_s(\mathbf{v},\varepsilon t')}{\partial\mathbf{v}}. \tag{A4}$$

In the above it is implicitly assumed that the real frequency is defined in the limit $\omega_{\mathbf{k}}=\lim_{\sigma\to 0}(\omega_{\mathbf{k}}+i\sigma)$ so that in the limit $t\to-\infty$ the integral is well defined.

If Eq. (A4) is expanded to first order in ε, then we obtain

$$\begin{aligned}\hat{f}_s(\mathbf{k},\mathbf{v},\varepsilon t)&=-\frac{e_s}{m_s}\int_{-\infty}^{t}dt'\,e^{-i(\omega_{\mathbf{k}}-\mathbf{k}\cdot\mathbf{v})(t'-t)}\times\left(1+(t'-t)\frac{\partial}{\partial t}\right)\hat{\mathbf{E}}(\mathbf{k},\varepsilon t)\cdot\frac{\partial F_s(\mathbf{v},\varepsilon t)}{\partial\mathbf{v}}\\&=-\frac{ie_s}{m_s}\left(\frac{1}{\omega_{\mathbf{k}}-\mathbf{k}\cdot\mathbf{v}}+i\frac{\partial}{\partial\omega_{\mathbf{k}}}\frac{1}{\omega_{\mathbf{k}}-\mathbf{k}\cdot\mathbf{v}}\frac{\partial}{\partial t}\right)\times\hat{\mathbf{E}}(\mathbf{k},\varepsilon t)\cdot\frac{\partial F_s(\mathbf{v},\varepsilon t)}{\partial\mathbf{v}}.\end{aligned} \tag{A5}$$

To derive the desired kinetic equation we consider

$$\frac{\partial F_s}{\partial t}=-\frac{e_s}{2m_s}\frac{1}{(2\pi)^3}\int d\mathbf{k}\left[\hat{\mathbf{E}}(\mathbf{k},\varepsilon t)\cdot\frac{\partial\hat{f}_s(-\mathbf{k},\mathbf{v},\varepsilon t)}{\partial\mathbf{v}}+\hat{\mathbf{E}}(-\mathbf{k},\varepsilon t)\cdot\frac{\partial\hat{f}_s(\mathbf{k},\mathbf{v},\varepsilon t)}{\partial\mathbf{v}}\right], \tag{A6}$$

where the equation has been symmetrized. In Eq. (A5) and thereafter we ignore the effect of slow time dependence because in Eq. (A6) terms on the right hand-side are of higher order. One may then obtain the following kinetic equation

$$\frac{\partial F_s}{\partial t}=\frac{e_s^2}{m_s^2}\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{\mathbf{k}}{|k|}\cdot\frac{\partial}{\partial\mathbf{v}}\left[\pi\delta(\omega_{\mathbf{k}}-\mathbf{k}\cdot\mathbf{v})\hat{E}_{\mathbf{k}}^2-\frac{1}{2}\frac{\partial}{\partial\omega_{\mathbf{k}}}\left(\mathcal{P}\frac{1}{\omega_{\mathbf{k}}-\mathbf{k}\cdot\mathbf{v}}\right)\frac{\partial}{\partial t}\hat{E}_{\mathbf{k}}^2\right]\frac{\mathbf{k}}{|k|}\cdot\frac{\partial F_s}{\partial\mathbf{v}}, \tag{A7}$$

where we have introduced the expressions $\hat{\mathbf{E}}(\mathbf{k},\varepsilon t)\cdot\hat{\mathbf{E}}^*(\mathbf{k},\varepsilon t)=\hat{E}_{\mathbf{k}}^2$ and have expressed the electrostatic field vector as $\hat{\mathbf{E}}(\mathbf{k},\varepsilon t)=\mathbf{k}\hat{E}_{\mathbf{k}}/|k|$.

In Eq. (A7) the term proportional to the delta function describes the resonant interaction whereas the term involving principal value represents the nonresonant wave-particle interactions. The term that represents the nonresonant wave-particle interaction is in agreement with that discussed in Refs. 11 and 17, whose derivations are more sophisticated. In Ref. 11 the analysis retains the use of the quantity gamma and in Ref. 17 a more formal analysis based From Fourier transform is used to discuss quasilinear theory with spontaneous emission. In the preceding discussion we deduce the following rule. That is, in order to include the nonresonant interactions we simply replace the delta function term by

$$\pi\delta(\omega_{\mathbf{k}}-\mathbf{k}\cdot\mathbf{v})E_{\mathbf{k}}^2\to\pi\delta(\omega_{\mathbf{k}}-\mathbf{k}\cdot\mathbf{v})E_{\mathbf{k}}^2-\frac{1}{2}\frac{\partial}{\partial\omega_{\mathbf{k}}}\left(\mathcal{P}\frac{1}{\omega_{\mathbf{k}}-\mathbf{k}\cdot\mathbf{v}}\right)\frac{\partial E_{\mathbf{k}}^2}{\partial t}.$$

This rule will be applied to the case when a quasilinear theory involving Alfvén waves is discussed.

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