Stochastic heating and acceleration of minor ions by Alfvén waves

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1. Introduction

The present paper addresses the stochastic heating of minor ions by obliquely-propagating low-frequency Alfvén waves in the solar wind. An important characteristics of the stochastic heating is unearthed by means of test particle simulation. That is, when the wave amplitude exceeds some threshold condition for stochasticity, the time-asymptotic kinetic temperature associated with the minor ions becomes independent of the wave amplitude, and it always approaches the value dictated by the Alfvén speed, to wit, $T_{\text{kin}} \sim m_i v_A^2/2$. During the course of the heating process the minor ions gain a net average parallel speed, $v_\parallel \sim v_A$, in the laboratory frame. These results are consistent with observations which find that minor heavy ions often move faster than the local protons with a speed roughly equal to the local Alfvén speed.


A number of theories that involve wave-particle interactions between Alfvén–cyclotron waves and ions in the solar corona and wind have been put forth [Marsch et al., 1982; Cranmer, 2001; Tu and Marsch, 2001; Gary et al., 2001; Vocks and Marsch, 2002; Hollweg and Isenberg, 2002]. A common thread in these theories is the resonant wave-particle interactions. However, there are theoretical difficulties with the application of the ion-cyclotron mechanism, and its role is not yet fully understood [Ofman, 2010]. Recently, the low-frequency Alfvén waves or kinetic Alfvén waves have attracted much attention. Several different theoretical approaches, which are not based on the cyclotron–resonant interaction, have been suggested [Chen et al., 2001; Voitenko and Goossens, 2004, 2006; Wang et al., 2006; Wu and Yang, 2006; Wu and Yoon, 2007; Nariyuki et al., 2010].

Two approaches deserve discussions in this regard. First, Wang et al. [2006], Wu and Yoon [2007], and Wang and Wu [2009] point out that the ions can get energized by Alfvén waves via non-resonant interactions. These authors demonstrate that the energy gained for the ions is proportional to $B_{\text{rms}}^2$, where $B_{\text{rms}}$ is the average magnetic field wave amplitude. It is found that the non-resonant process leads to an efficient pitch-angle scattering of ions [Wu and Yang, 2007; Wu et al., 2009; Yoon et al., 2009]. This non-resonant heating mechanism is also applied to calculate the heating rate of ions by turbulent Alfvén waves in the solar upper chromosphere and transition region considering the ionization and recombination process [Wang and Wu, 2009].

The second approach imposes a somewhat different concept of heating by emphasizing stochasticity of particle motion. Theoretical formulation along this line is based on nonlinear dynamics. In a number of articles effects of large amplitude Alfvén wave with oblique propagation on a single particle motion is discussed by Chen and his colleagues [Chen et al., 2001; White et al., 2002; Guo et al., 2008; Lu and Chen, 2009]. It is found that when the wave amplitude exceeds certain threshold value the motion of a particle may change from regular to stochastic due to high order resonance, which can take place at a fraction of the ion cyclotron frequency. Their discussions emphasize the threshold of wave amplitude that can lead to stochasticity. However, it is unclear how much energy the ions can gain from the waves and how it is related to the heating process in the eyes of layman. We are interested in the physical relation between heating and stochasticity.

In the following we shall introduce a concept of “stochastic heating” for which a kinetic temperature may be defined. As we shall revisit the problem of stochastic heating by oblique Alfvén waves by emphasizing the pitch-angle scattering and the quasi-asymptotic heating of the ions. We thus attempt to shed a new light on this problem. As we shall discuss subsequently in more detail, one of the most important findings is that the asymptotic temperature of the ions is independent of the wave amplitude $B_{\text{rms}}$, as long as the amplitude is above the threshold of stochasticity as expounded by Chen and his colleagues. This new finding is significant in view of the fact that the customary non-resonant heating discussed by Wu and Yoon [2007] and
magnetic field; $B_0$ and $\varphi_0$ are the wave amplitude and random phase for mode $k$; $v_\perp$ is the Alfvén speed; $\omega = k_v v_\perp$ is the wave frequency; $\mathbf{l}_y$ is an unit vector. The particle dynamics is governed by

$$\frac{m}{v} \frac{dv}{dt} = q \mathbf{v} \times (B_0 + B_0'), \quad \frac{dr}{dt} = \mathbf{v}. \quad (2)$$

Where $B_0$ is the ambient magnetic field in the $z$ direction. The equations of motion are solved with Boris algorithm, where the kinetic energy of the particle is conserved in the calculation over the cyclotron motion. We discretize the Alfvén wave spectrum as follows: $\omega_j = \omega_1 + (j-1)\Delta \omega$ $(j = 1,2,...,N)$, where $\Delta \omega = (\omega_N - \omega_1)(N - 1)$, $N$ is the total number of wave modes. The amplitude of each wave mode is considered to be equal to each other, and is a constant. The time is normalized with respect to the proton gyro period, and the time step is $\Omega_{p,t} = 0.001$, where $\Omega_{p,t}$ is the proton cyclotron frequency.

We are interested in the minor ions, so we adopt $He^+$ as the test particles. The total number of test particles is $10^9$, that are initially distributed at random during the time interval $\Omega_{p,t} = [0,2\pi]$ and along the spatial range $x\Omega_{p,t}/v_\perp = [0,2000]$ and $z\Omega_{p,t}/v_\perp = [0,2000]$. Their initial velocities are assumed to be distributed according to Maxwellian form with thermal speed $v_T = 0.05 v_\perp$ in the laboratory frame, which is less than $v_\perp$ to ensure that the cyclotron resonance condition cannot be satisfied.

3. Heating and Acceleration of Minor Ions

Chen and his colleagues [Chen et al., 2001; White et al., 2002; Guo et al., 2008] discuss the transition from regular to stochastic orbits by means of Poincaré plot for a monochromatic wave. They suggested that once the island overlapping condition $k_v B_0/v_\perp \Omega_{p,t}$ is of order 0.1 or so, the particle orbit makes a transition from periodic to stochastic orbit. We thus likewise also investigate the Poincaré section, making use of the monochromatic wave first (i.e., mode number $N = 1$). The Poincaré plot facilitates the distinction between regular and stochastic particle orbits. Here we plot points corresponding to $v_y = 0$ and $v_y > 0$. Figure 1 shows the Poincaré plot at different times. The plus signs represent the last 400 points recorded near the end of the simulation time. The parameters are $\alpha = 45^\circ$, $B_0^2/B_0'^2 = 0.175$, and $\omega = 0.15 \Omega_{p,t}$.

Wang et al. [2006] dictates that the thermal energy gain is proportional to $B_0^2$. We find that the stochastic heating is determined by the Alfvén speed, as will be shown. This realization may be helpful for understanding why the observation shows that minor ions flow faster than the protons by approximately the local Alfvén speed. In the remainder of this Letter, we discuss our findings systematically.

2. Physical Model

We assume that intrinsic Alfvén waves pervade the solar corona and interplanetary space. We assume that the presence of minor ions do not have any significant impact on the intrinsic turbulence. Consequently, we adopt the test particle simulations. Let us consider linearly polarized incoherent Alfvén waves propagating obliquely with respect to the ambient magnetic field. In the reference frame moving with the wave speed $v_\perp$, the wave magnetic field vector can thus be expressed as

$$B_n = \sum_{k,y} B_k \cos \psi_1 \mathbf{l}_y, \quad (1)$$

where $\psi_1 = k_x x + k_z z + \varphi_k$ and $\tan \alpha = k_x/k_z$; $k_x$ and $k_z$ are wave numbers perpendicular and parallel to the ambient wave at (a) $\Omega_{p,t} = 1000$ and (b) $\Omega_{p,t} = 10000$. Plus signs are the last 400 points recorded near the end of the simulation. The input parameters are $\alpha = 45^\circ$, $B_0^2/B_0'^2 = 0.175$, and $\omega = 0.15 \Omega_{p,t}$.

Figure 1. Poincaré plot for a monochromatic Alfvén wave at (a) $\Omega_{p,t} = 1000$ and (b) $\Omega_{p,t} = 10000$. Plus signs are the last 400 points recorded near the end of the simulation. The input parameters are $\alpha = 45^\circ$, $B_0^2/B_0'^2 = 0.175$, and $\omega = 0.15 \Omega_{p,t}$.
The time evolution of the kinetic temperature in $=3.0$, $=0.175$, case C is for $=0.10$, $=0.0025$, case B is for $=0.20$, and case D is $t$. The kinetic temperature is defined by $=\langle v^2 \rangle$, where the bracket $\langle \cdot \rangle$ denotes an average over all particles. Here, the bracket $\langle \cdot \rangle$ denotes an average over all particles. Figure 2 shows $T_{\text{kin}}$ versus time, for four different wave amplitudes: (A) $B_{\text{w}}^2/B_0^2 = 0.0025$, (B) $B_{\text{w}}^2/B_0^2 = 0.175$, (C) $B_{\text{w}}^2/B_0^2 = 0.20$, and (D) $B_{\text{w}}^2/B_0^2 = 0.25$. Case A does not satisfy the threshold of stochastic heating, while cases B, C and D exceed the threshold.

Here we present a discussion. The saturation of ion kinetic energy, when the wave amplitude reaches a threshold value, may be explained physically. This is the result of energy conservation in the wave frame. The solution to equation (2) satisfies the condition [Wu et al., 1997]:

$$v^2(t) + [v_i(t) - v_A]^2 = v^2(0) + [v_i(0) - v_A]^2,$$

where $v_A$ and $v_i$ stands for velocity components perpendicular and parallel to the ambient magnetic field. The motion of ions on the spherical surface in velocity space, defined by equation (3), is attributed to pitch-angle scattering. This picture is consistent with the non-resonant heating by Alfvén waves discussed by Wang et al. [2006], Wu and Yoon [2007], and Yoon et al. [2009]. During the process the Alfvén waves are pitch-angle scattering the ions to form a spherical shell distribution in the wave frame. When the “threshold” value of the wave amplitude is reached, the velocity distribution becomes a complete sphere, that is $f(v) = \delta(v - v_A)$ if initially the ions are at rest with cold temperature in the laboratory frame, which implies a kinetic temperature $T_{\text{kin}} = \frac{\mu}{2} \int f(v) v^2 d^3v = \frac{\mu m_i v_A^2}{2}$. This also implies that ions gain a bulk parallel velocity equal to the Alfvén speed, because the center of the sphere shell is situated at the Alfvén velocity in the laboratory frame. We define this stage of heating as “stochastic heating” because the result is compatible to non-resonant heating and discussion of stochasticity. However, as we shall discuss next, the main difference is that the present stochastic heating mechanism is largely independent of the wave amplitude $B_{\text{w}}$, as long as it exceeds the threshold. In contrast, the non-resonant heating is proportional to $B_{\text{w}}^2$. To elaborate this point we present Figures 2 and 3 which are explained as follows.

Figure 1 was obtained with a monochromatic Alfvén wave. However, because of the non-dispersive nature of the Alfvén wave, the main results are still applicable in the situation of a spectrum of Alfvén waves. To validate this, we now discuss the simulations with incoherent Alfvén waves. Figure 2 shows the time evolution of the ion kinetic temperature normalized with respect to $m_i v_A^2/2$, versus normalized time $\Omega_{p,t}$, for the following physical configuration: the wave mode number $N = 30$, $\omega_1 = 0.05 \Omega_{p,t}$, $\omega_N = 0.10 \Omega_{p,t}$, and for $He^+$ ions. The kinetic temperature is defined by $T_{\text{kin}} = \frac{\mu m_i}{2} \langle v^2 \rangle + \frac{\mu}{2} \langle v_2^2 \rangle$. Here, the bracket $\langle \cdot \rangle$ denotes an average over all particles. Figure 2 shows $T_{\text{kin}}$ versus time, for four different wave amplitudes: (A) $B_{\text{w}}^2/B_0^2 = 0.0025$, (B) $B_{\text{w}}^2/B_0^2 = 0.175$, (C) $B_{\text{w}}^2/B_0^2 = 0.20$, and (D) $B_{\text{w}}^2/B_0^2 = 0.25$. Case A does not satisfy the threshold of stochastic heating, while cases B, C and D exceed the threshold.

Figure 3. Plot of average parallel velocity in the laboratory frame associated with the ions corresponding to the simulation shown in Figure 2.
amplitude exceeds the threshold, the asymptotic results are independent of the wave amplitude and the wave frequency. [13] Figure 3 shows the same test-particle simulation result as shown in Figure 2, except that we plot the result in terms of average velocity associated with the ions. We see that average \( v_1 \) approaches \( v_A \) at the end of the simulation. Note that Figure 3 is in the laboratory frame. In velocity space, the center of spherical shell distribution is \( v_A \), and thus, the ions parallel velocity should lie in the range \( 0v_A \) to \( 2v_A \). Over the time range of the present simulation, the ions attained bulk parallel speed roughly equal to \( v_A \), as been shown in Figure 3.

[14] This finding has an important ramification for observation. As mentioned in the Introduction, heavy ions are detected in the solar wind to possess flow speeds that are faster than the proton speed by roughly the local Alfvén speed. The present finding is consistent with this observation. According to the present analysis, the heavy ions should be picked up by Alfvén waves, thus forming a quasi-isotropic spherical shell velocity distribution function that has a net parallel speed equal to the local Alfvén speed. Although the direct observation of heavy ion distribution is difficult, we believe that the observed differential speed of heavy ions versus the protons is a circumstantial evidence that the present stochastic heating mechanism may be operative.

[15] White et al. [2002] and Lu and Chen [2009] find that the threshold value for the ion stochasticity in the case of a spectrum of Alfvén waves is substantially lower than that for the monochromatic Alfvén wave. We thus believe that in the solar corona as well as the solar wind where Alfvén waves exist pervasively, the said threshold condition may be satisfied. Observational results have shown that the waves fields can often have energy density comparable to that of the unperturbed magnetic field in solar wind [Belcher and Davis, 1971].

[16] Finally, recent observations have shown that kinetic Alfvén waves, the electrostatic field along the ambient magnetic field can play significant roles for the heating and acceleration of charged particles [Voitenko and Goossens, 2006; Wu and Yang, 2006]. Assuming the proton temperature is equal to the ion temperature, \( T_p = T_i \), and using \( k_p = k_i, \omega = k_i v_A = 0.1 \Omega_i, \) and \( v_{pi} = 0.05 v_A \), we find that \( k_p \rho_p = 0.005 \), where \( \rho_p = \sqrt{2}/v_p \) is the proton gyroradius. This indicates that ignoring the kinetic effect is justifiable in this paper. For hotter plasmas kinetic treatment may be necessary, which is beyond the scope of the present manuscript. Our assumption may be more appropriate in the inner heliosphere, closer to the sun.

4. Conclusions

[17] The notion of stochastic heating by obliquely propagating Alfvén waves was put forth in a number of papers [Chen et al., 2001; White et al., 2002; Guo et al., 2008]. In these papers, the authors demonstrated that particle orbits undergo a transition from regular to random orbits for an obliquely propagating Alfvén wave, when the wave amplitude exceeds a certain threshold value, thus indicating ion heating. However, the above references emphasize the detailed nonlinear dynamics of the single particle orbits, and as such, systematic analyses of actual ion heating was not explored by these authors.

[18] In the present paper, we have re-analyzed the stochastic heating theory in a new light. Analyzing the single particle orbit by means of Poincaré section plot, as is done in the above references, is not so useful if our aim is to discuss the heating and acceleration of ions by the said mechanism. We thus chose to display the test-particle simulation result in terms of the time history of the total ion energy. We have thus uncovered an important characteristics of the stochastic heating. That is, as long as the low-frequency obliquely-propagating Alfvén wave exceeds the threshold amplitude, the quasi-asymptotic kinetic temperature associated with the minor ions is independent of the wave amplitude or its characteristic wave frequency, and it always approaches the value dictated by the Alfvén speed, \( T_{kin} \sim m_A v_A^2/2 \).

[19] The physical mechanism for the asymptotically independent heating is the pickup process that involves the formation of spherical shell velocity distribution function and the pitch-angle scattering. This process is equivalent to the minor ions gaining a net average parallel speed, \( v_{\parallel} \sim v_A \) in the laboratory frame. These findings are highly relevant to the observed kinetic properties of heavy solar wind ions from Ulysses [von Steiger et al., 1995; von Steiger and Zurbuchen, 2006], that is, the mass-proportional kinetic temperatures and the similar average values of the bulk speed for different kind of heavy ions, especially that the bulk speed is roughly equal to the local Alfvén speed.

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