Angular momentum transport in a multicomponent solar wind with differentially flowing, thermally anisotropic ions

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ABSTRACT

Context. The Helios measurements of the angular momentum flux \( L \) of the fast solar wind lead to a tendency for the fluxes associated with individual ion angular momenta of protons and alpha particles, \( L_p \) and \( L_\alpha \), to be negative (i.e., in the sense of counter-rotation with the Sun). However, the opposite holds for the slow wind, and the overall particle contribution \( L_P = L_p + L_\alpha \) tends to exceed the magnetic contribution \( L_M \). These two aspects are at variance with previous models.

Aims. We examine whether introducing realistic ion temperature anisotropies can resolve this discrepancy.

Methods. From a general set of multifluid transport equations with gyrotropic species pressure tensors, we derive the equations governing both the meridional and azimuthal dynamics of outflows from magnetized, rotating stars. The equations are not restricted to radial flows in the equatorial plane but valid for general axisymmetric winds that include two major ion species. The azimuthal dynamics are examined in detail, using the empirical meridional flow profiles for the solar wind, constructed mainly according to measurements made in situ.

Results. The angular momentum flux \( L \) is determined by the requirement that the solution to the total angular momentum conservation law is unique and smooth in the vicinity of the Alfvén point, defined as where the combined Alfvénic Mach number \( M_F = 1 \). \( M_F \) has to consider the contributions from both protons and alpha particles. Introducing realistic ion temperature anisotropies may introduce a change of up to 10% in \( L \) and up to \( \sim 1.8 \text{ km s}^{-1} \) in azimuthal speeds of individual ions between 0.3 and 1 AU, compared with the isotropic case. The latter has strong consequences on the relative importance of \( L_p \) and \( L_M \) in the angular momentum budget.

Conclusions. However, introducing ion temperature anisotropies cannot resolve the discrepancy between in situ measurements and model computations. For the fast-wind solutions, while in extreme cases \( L_p \) may become negative, \( L_\alpha \) never does. On the other hand, for the slow solar wind solutions examined, \( L_p \) never exceeds \( L_M \), even though \( L_M \) may be less than the individual ion contribution, since \( L_p \) and \( L_\alpha \) always have opposite signs for the slow and fast wind alike.

Key words. Sun: rotation – Sun: magnetic fields – solar wind – stars: rotation – stars: winds, outflows

1. Introduction

The angular momentum loss of a rotating star due to its outflow influences the rotational evolution of the star considerably, and is therefore of astrophysical significance in general (see e.g., Weber & Davis 1967; Belcher & MacGregor 1976; Mestel & Spruit 1987; Bouvier et al. 1997). However, direct tests of in situ measurements against theories such as those presented by Weber & Davis (1967) are only possible for the present Sun. A substantial number of studies have been conducted and were compiled in the comprehensive paper by Pizzo et al. (1983), who themselves paid special attention to the Helios measurements of specific angular momentum fluxes. The measurements, further analyzed by Marsch & Richter (1984), are unique in that they allow the individual ion contribution from protons \( L_p \) and alpha particles \( L_\alpha \) to the solar angular-momentum loss rate per steradian \( L \) to be examined. For instance, despite the significant scatter, the data exhibit a distinct trend for \( L_p \) to be positive (negative) for solar winds with proton speeds \( v_p \) below (above) 400 km s\(^{-1}\). A similar trend for \( L_\alpha \) is also found on average. The magnetic contribution \( L_M \), on the other hand, is remarkably constant. A mean value of \( L_M = 1.6 \times 10^{29} \text{ dyne cm s}^{-1} \) can be quoted for the solar winds of all flow speeds and throughout the region from 0.3 to 1 AU. For comparison, the mean values of angular momentum fluxes carried by ion flows in the slow solar wind are \( L_p = 19.6 \) and \( L_\alpha = 1.3 \times 10^{29} \text{ dyne cm s}^{-1} \) (see Table II of Pizzo et al. 1983). The overall particle contribution to \( L \) is then \( L_P = L_p + L_\alpha = 20.9 \times 10^{29} \text{ dyne cm s}^{-1} \), which tends to be larger than \( L_M \). It is noteworthy that a more recent study by Scherer et al. (2001) showed how examining the long-term variation of the non-radial components of the solar wind velocity and the corresponding angular momentum fluxes can help us understand the heliospheric magnetic field better.

Alpha particles should be placed on the same footing as protons from the perspective of solar wind modeling, given their non-negligible abundance and the fact that there tends to exist a substantial differential speed \( v_{\alpha p} \equiv |v_\alpha| - |v_p| \). As shown by the Helios measurements, a \( v_{\alpha p} \) amounting to up to 20–30% of the local proton speed may occur in both the fast and slow solar winds (Marsch et al. 1982a,b), with the latter being exemplified by an event that took place on day 117 of 1978, when a positive \( v_{\alpha p} \sim 100 \text{ km s}^{-1} \) was found at 0.3 AU (Marsch et al. 1981). That on the average \( v_{\alpha p} = 0 \) in the slow wind simply reflects that the events with positive and negative \( v_{\alpha p} \) occur with nearly equal frequency (Marsch et al. 1982a). As for the alpha abundance relative to protons, a value of 4.6% (0.4–10%) is well-established for the fast (slow) solar wind (e.g., McComas et al. 2000). Therefore alpha particles can play an important role as far as the energy and linear momentum balance of the solar wind are concerned. When it comes to the problem of angular
momentum transport, it was shown that in interplanetary space not only the angular momentum flux carried by the alpha particles \( L_a \), but also that convected by the protons \( L_p \) are determined by the terms associated with \( \nu_{\text{pro}} \) (Li & Li 2006). This essentially derives from the requirement that the proton-alpha velocity difference vector be aligned with the instantaneous magnetic field. As a consequence, these terms have no contribution to the overall angular momentum flux convected by the ion flow \( L_\parallel \), which turns out to be smaller than \( L_\perp \) in all the models examined in the parameter study by Li et al. (2007). This, together with the fact that \( L_p \) is always positive (i.e., in the sense of corotation with the Sun), is at variance with the Helios measurements.

A possible means to reconcile the measurements and the model computation is to incorporate the species temperature anisotropies. This is because the total pressure tensor \( P = \sum \rho_i \mathbf{p}_i \sum \rho_i \mathbf{p}_i \) summed over all species \( s \) participates in the problem of angular momentum transport via the component \( P_\parallel = P\parallel - P\perp \) where \( \parallel \) and \( \perp \) are relative to the magnetic field \( \mathbf{B} \) (see e.g., Weber 1970, hereafter referred to as W70). While the overall loss rate per steradian \( L_\parallel \) may not be significantly altered, the azimuthal speed of the solar wind and therefore the particle part of \( L_\parallel \) may be when compared with the isotropic case. Note that in the treatment of W70 the solar wind was seen as a bulk flow and the ion species are not distinguished. On the other hand, the formulation by Li & Li (2006) did not take into account the pressure anisotropy, which is a salient feature of the velocity distribution functions for both protons and alpha particles as revealed by the Helios measurements (Marsch et al. 1982a,b). It therefore remains to be seen how introducing the pressure anisotropy influences individual ion azimuthal speeds. Moreover, the simple, prescribed functional form for \( P_\parallel \) assumed in W70 needs to be updated in light of the more recent particle measurements.

The aim of the present paper is to extend the W70 study in three ways. First, we shall follow a multicomponent approach and examine the angular momentum transport in a solar wind comprising protons, alpha particles and electrons where a sub- and examine the angular momentum transport in a solar wind comprising protons, alpha particles and electrons where a sub-

2. Mathematical formulation

Presented in this section is the mathematical development of the equations that govern the angular momentum transport in a time-independent solar wind which consists of electrons (e), protons (p) and alpha particles (α). Each species \( s \) (\( s = e, p, \alpha \)) is characterized by its mass \( m_s \), electric charge \( e_s \), number density \( n_s \), mass density \( \rho_s \), velocity \( \mathbf{v_s} \), and partial pressure \( p_s \). If measured in units of the electron charge \( e \), \( e_s \) may be expressed by \( e_s = Z_se_e \) with \( Z_s = -1 \) by definition.

To simplify the mathematical treatment, a number of assumptions have been made and are collected as follows:

1. Symmetry about the magnetic axis is assumed, i.e., \( \partial / \partial \phi = 0 \) in a heliocentric spherical coordinate system \( \mathbf{r} = (r, \theta, \phi) \).

2. The velocity distribution function (VDF) of each species is close to a bi-Maxwellian, and the pressure tensor is gyrotropic, i.e., \( p_s = p_s^{\parallel} + (p_s^{\perp} - p_s^{\parallel})^\perp B_b \), where \( B_b \) is the unit dyad and \( b \) is the unit vector along the magnetic field \( \mathbf{B} \). The temperatures pertaining to the degrees of freedom parallel and perpendicular to \( \mathbf{B} \) follow from the relation \( p_s^{\parallel} = n_s k_B T_s^{\parallel} \), where \( k_B \) is the Boltzmann constant.

3. Quasi-neutrality is assumed, i.e., \( n_e = \sum n_s n_k, \) \( n_k = (k = p, \alpha) \), except when the reduced meridional momentum equation is derived.

2.1. Multi-fluid equations

The equations appropriate for a multi-component solar wind plasma with gyrotropic species pressure tensors may be found by neglecting the electron inertia \( (m_e \equiv 0) \) in the equations given by Barakat & Schunk (1982). Following the same procedure as given in the Appendix A.1 in Li & Li (2006), one may find

\[
\nabla \cdot (n_e \mathbf{v}_e) = 0, \quad (1)
\]

\[
\mathbf{v}_e \cdot \nabla p_e + \frac{Z_e \nabla \cdot \mathbf{p}_e}{n_e m_e} + \frac{G M_\odot}{r^2} = 0, \quad (2)
\]

\[
\frac{1}{n_s m_s} \frac{\delta M_s}{\delta t} = \mathbf{Z}_s \frac{n_s m_s}{n_e} \frac{\delta p_s}{\delta t} = \frac{Z_s}{4 \pi m_e n_e} (\nabla \times \mathbf{B}) \times \mathbf{B} + Z_e n_e Z_i \left( \mathbf{v}_i - \mathbf{v}_e \right) \times \mathbf{B} = 0, \quad (3)
\]

\[
\nabla \cdot (n_e \mathbf{v}_e) = 0, \quad (4)
\]

\[
\nabla \cdot (n_\ell \mathbf{v}_\ell) = 0, \quad (5)
\]

where the subscript \( \ell \) refers to all species \( s = e, p, \alpha \), while \( k \) stands for ion species only \( (k = p, \alpha) \). The gravitational constant is denoted by \( G \), \( M_\odot \) is the mass of the Sun, and \( c \) is the speed of light. The momentum and energy exchange rates due to the Coulomb collisions of species \( s \) with the remaining ones are denoted by \( \delta E_\ell / \delta t \) and \( \delta E_\ell^{\perp} / \delta t \), respectively. The third-rank tensor \( \mathbf{Q}_s \), together with the heat flux vectors \( \mathbf{q}_s^{\parallel} \) associated with parallel and perpendicular degrees of freedom, arises

\[
\nabla \cdot \mathbf{Q}_s = \frac{1}{2} \left( \mathbf{Q}_s \cdot \nabla \mathbf{B} \right), \quad (6)
\]

\[
\nabla \cdot (\mathbf{v}_e \times \mathbf{B}) = 0, \quad (7)
\]

where the subscript \( e \) refers to all species \( s = e, p, \alpha \), while \( k \) stands for ion species only \( (k = p, \alpha) \). The gravitational constant is denoted by \( G \), \( M_\odot \) is the mass of the Sun, and \( c \) is the speed of light. The momentum and energy exchange rates due to the Coulomb collisions of species \( s \) with the remaining ones are denoted by \( \delta E_\ell / \delta t \) and \( \delta E_\ell^{\perp} / \delta t \), respectively. The third-rank tensor \( \mathbf{Q}_s \), together with the heat flux vectors \( \mathbf{q}_s^{\parallel} \) associated with parallel and perpendicular degrees of freedom, arises
from the deviation of species VDFs from an exact bi-Maxwellian (Baratok & Schunk 1982). Moreover, \( H^{\parallel \perp} \) stands for the heating rates applied to species \( s \) in the parallel and perpendicular directions from some non-thermal processes. They may be determined by assuming that the heating derives from the dissipation of Alfven-ion cyclotron waves (e.g., Hollweg & Isenberg 2002), or more simply in some ad hoc fashion such as employed in Leer & Axford (1972). The operators \( \nabla_{\parallel} \) and \( \nabla_{\perp} \) are defined by

\[
\nabla_{\parallel} = \frac{\partial}{\partial \xi_{\parallel}} = \frac{\partial}{\partial \xi_{\parallel}} ~ \text{and} ~ \nabla_{\perp} = \nabla - \nabla_{\parallel},
\]

respectively.

In Eq. (2), the subscript \( j \) stands for the ion species other than \( k \), namely, \( j = p \) for \( k = \alpha \) and vice versa. As can be seen, in addition to the term \((\nabla \times B) \times B\), the Lorentz force possesses a new term in the form of the cross product of the ion velocity difference and magnetic field. Physically, this new term represents the mutual gyration of one ion species about the other, the axis of gyration being in the direction of the instantaneous magnetic field. Furthermore, Eq. (5) is the time-independent version of the magnetic induction law, which states that the magnetic field is frozen in the electron fluid. It may be readily shown that the effects of the electron pressure gradient, the Hall term, and the momentum exchange rates as contained in the generalized Ohm’s law can be safely neglected given the large spatial scale in question (a formal evaluation of the different terms can be found in Sect. 2.1 of Li et al. 2006).

To proceed, we choose a flux tube coordinate system, in which the base vectors are \( \hat{e}_{\parallel}, \hat{e}_{\perp}, \hat{e}_{\phi} \), where

\[
\hat{e}_{\parallel} = B_{P}/|B_{P}|, \quad \hat{e}_{\perp} = \hat{e}_{\phi} \times \hat{e}_{\parallel},
\]

with the subscript \( P \) denoting the poloidal component. Moreover, the independent variable \( l \) is the arclength along the poloidal magnetic field line measured from its footpoint at the Sun. This choice permits the decomposition of the magnetic field and species velocities as follows,

\[
B = B_{P}\hat{e}_{\parallel} + B_{\phi}\hat{e}_{\phi}, \quad \mathbf{v}_{s} = v_{s\parallel}\hat{e}_{\parallel} + v_{s\perp}\hat{e}_{\perp} + v_{s\phi}\hat{e}_{\phi},
\]

where \( s = e, p, \alpha \). From the assumption of azimuthal symmetry, and the assumption that the solar wind is time-independent, one can see from the poloidal component of Eq. (5) that \( B_{P} \hat{e}_{\parallel} \) should be strictly in the direction of \( B_{P} \). In other words, \( v_{s\parallel} = 0 \) to a good approximation. Now let us consider the \( \phi \) component of the momentum Eq. (2). Since the frequencies associated with the spatial dependence are well below the ion gyro-frequency \( \Omega_{i} = (zeB) / (mc) \) (\( k = p, \alpha \)), from an order-of-magnitude estimate one can see that \( |v_{p\parallel} - v_{\alpha\parallel}| \ll |v_{\phi\parallel}| \). Combined with the fact that \( v_{\phi\parallel} = 0 \), this leads to that both \( v_{p\parallel} \) and \( v_{\alpha\parallel} \) should be very small and can be safely neglected unless they appear alongside the ion gyro-frequency. With this in mind, one can find from the \( N \) component of Eq. (2) that

\[
v_{\phi\alpha} - v_{\phi\beta} = \frac{B_{\perp}}{B_{P}} (v_{p\alpha} - v_{p\beta}).
\]

That is, the ion velocity difference is strictly aligned with the magnetic field. This alignment condition further couples one ion species to the other.

The fact that \( v_{\phi\beta} (s = e, p, \alpha) \) is negligible means that the system of vector equations may be decomposed into a force balance condition across the poloidal magnetic field and a set of transport equations along it. In the present paper, however, we simply replace the force balance condition by prescribing an analytical meridional magnetic field configuration. Moreover, we examine in detail only the azimuthal dynamics, leaving a brief discussion on the poloidal one in the Appendix.

Fig. 1. Adopted meridional magnetic field configuration in the inner corona. Here only a quadrant is shown in which the magnetic axis points upward, and the thick contours labeled \( F \) and \( S \) delineate the lines of force along which the fast and slow solar wind solutions are examined, respectively. Also shown is how to define the geometrical factor \( R \) and the base vectors \( \hat{e}_{\parallel}, \hat{e}_{N} \) and \( \hat{e}_{\phi} \) of the flux tube coordinate system (see Sect. 2).

2.2. Azimuthal dynamics

The \( \phi \) component of the magnetic induction law (5) gives

\[
\nabla \cdot \left( \frac{1}{R} \left( B_{\phi}v_{\phi\perp} - v_{\phi\parallel} B_{P} \right) \right) = 0.
\]

Now that \( v_{\phi\perp} = v_{\phi\parallel} \hat{e}_{\phi} \), one may readily integrate Eq. (8) along a magnetic line of force to yield

\[
v_{\phi\parallel} = \frac{A_{\perp} R + B_{\phi}}{B_{\parallel}} v_{\phi\parallel}.
\]

Here \( R = r \sin \theta \) is a geometrical factor to be evaluated along a given line of force (see Fig. 1), and \( A_{\perp} \) is a constant of integration and should be identified as the angular rotation rate of the footpoint of the magnetic flux tube. Taking into account the alignment condition (7), one may find that

\[
v_{\phi\alpha} = \frac{A_{\perp} R + B_{\phi}}{B_{\parallel}} v_{\phi\parallel}.
\]

where \( s = e, p, \alpha \). Therefore in a frame of reference that corotates with the Sun, the velocities of all species are aligned with the magnetic field.

Another equation that enters into the azimuthal dynamics is the \( \phi \) component of the total momentum. In the present case, it reads

\[
\frac{1}{R} \left[ \sum_{k} \rho_{k} v_{\phi k} \left( R v_{\phi k} \right) - \frac{B_{\parallel}}{4\pi} \left( 1 - \frac{4\pi P_{\phi}}{B^{2}} \right) R B_{\phi} \right] = 0,
\]

where

\[
P_{\phi} = P_{\parallel} - P_{\perp}, \quad P_{\perp} = \sum_{s} P_{s\parallel}^{\perp},
\]

and the prime \( \prime = \hat{e}_{\parallel} \cdot \nabla \) is the directional derivative along the poloidal magnetic field.
For a time-independent flow $\rho_t v_{\phi}/B_l = \text{const}$. It then follows that

$$R \left[ \psi_{\phi} + \eta v_{\phi} - \frac{B_l B_0}{4 \pi \rho_{t} v_{\phi}^2} \left( 1 - \frac{4 \pi \rho_{t} v_{\phi}^2}{B_l^2} \right) \right] = A_L,$$  \hspace{1cm} (13)

where the constant $\eta = (\rho_{t} v_{\phi})/\rho_{t} v_{\phi}^2$ is the ion mass flux ratio, and $A_L$ is a constant of integration. Physically, $A_L$ is related to the angular momentum loss rate per steradian $L$ by

$$L = M_p A_L,$$  \hspace{1cm} (14)

where

$$M_t = \rho_{t} v_{\phi} B_E E_2 / B_l E_1,$$  \hspace{1cm} (15)

is the ion mass loss rate per steradian scaled to the Earth orbit $r_E = 1$ AU, with $B_E$ denoting the strength of the poloidal magnetic field at $r_E$. It follows that the angular momentum loss rate of the Sun due to the solar wind $\dot{L}$ of colatitude. Equation (13) shows that $L$ consists of the contributions due to individual ion angular momenta $L_a$, the magnetic stresses $L_M$ and the total pressure anisotropy $L_{an}$, where

$$[L_a, L_M, L_{an}] = R \left( B_E E_2 / B_l E_1 \right) \left[ \rho_{t} v_{\phi}^2, -B_l B_0 / 4 \pi \rho_{t} v_{\phi}^2, B_l B_0 / 4 \pi \rho_{t} v_{\phi}^2 \right].$$  \hspace{1cm} (16)

with $k = p, \alpha$.

Substituting Eqs. (10) into (13), one may find

$$\tan \Phi \left[ M_t^2 - \left( 1 - \beta^4 \cos^2 \Theta \right) \right] = \epsilon \left[ A_L / (1 + \eta) A_L R^2 \right] - 1,$$  \hspace{1cm} (17)

where $\tan \Phi = B_\phi / B_l$ defines the magnetic azimuthal angle $\Phi$, and

$$M_t^2 = M_p^2 + M_\alpha^2, M_t^2 = 4 \pi \rho_{t} v_{\phi}^2 B_l E_2 / B_l E_1 (k = p, \alpha),$$

$$\beta^4 = \frac{4 \pi \rho_{t} v_{\phi}^2}{B_l E_1}, \epsilon = 1 + \eta M_t^2 / \psi_{\psi}.$$  \hspace{1cm} (18)

By definition, $M_t$ is the combined poloidal Alfvénic Mach number, which involves both ion species. For a typical solar wind, between 1 $R_\odot$ and 1 AU there exists a point where $M_t = 1$, which is to be called the Alfvén point and denoted by $r_a$. As discussed in detail by Li & Li (2006), when species temperature anisotropy is absent ($P_\alpha = 0$ and therefore $\beta^4 = 0$), for Eq. (17) to possess a solution that passes smoothly through $r_a$ the two constants $A_L$ and $A_\Theta$ have to be related by

$$A_L = (1 + \eta) \lambda R^2 R^2,$$  \hspace{1cm} (19)

where the subscript $a$ denotes quantities evaluated at the Alfvén point. When $\beta^4$ is not zero, a direct relation between $A_L$ and $A_\Theta$ is not as obvious since now Eq. (17) becomes cubic in $\tan \Phi$. Nevertheless, one may write $A_L$ as $A_L = \lambda A_{10}$, where $A_{10}$ is determined through Eq. (19) and therefore $\lambda$ stands for the correction due to a finite $\beta^4$. It then follows that

$$c_3 \tan \Theta + c_2 \tan \Phi + c_1 \tan \Phi + c_2 = 0,$$  \hspace{1cm} (20)

where

$$c_3 = M_t^2 - 1, c_2 = \epsilon \left[ 1 - \lambda \left( \frac{R_{10}}{R} \right)^2 \right],$$

$$c_1 = M_t^2 - 1 + \beta^4.$$  \hspace{1cm} (21)

Given the meridional flow profiles along a prescribed magnetic field line, Eq. (20) possesses only one real root at locations far away from $r_a$. However, in the vicinity of $r_a$, there exists in general three real roots and they diverge near $r_a$. The requirement that there exists a unique solution that is smooth from 1 $R_\odot$ out to 1 AU determines $\lambda$ (Weber & Davis 1970; Weber 1970).

### 3. Meridional magnetic field and flow profiles

In principle, one needs to solve Eqs. (A.1) to (A.4) together with Eq. (17) simultaneously to gain a quantitative insight. In the present paper, we refrain from doing so because from previous experience it proves difficult to yield the flow profiles that satisfactorily reproduce in situ measurements such as made by Helios. Take the proton-alpha speed difference $v_{\alpha}$ in the fast solar wind for example. It is observationally established that $v_{\alpha}$ closely tracks the local Alfvén speed in the heliocentric range $r > 0.3$ AU (Marsch et al. 1982a). So far this fact still poses a theoretical challenge: adjusting the ad hoc heating parameters, or fine-tuning the cyclotron resonance mechanism is unable to produce such a behavior (see, e.g. Hu & Habbal 1999). We therefore adopt an alternative approach by prescribing the background meridional flow profiles that mimic the observations and then examining what consequences the species anisotropies have on the azimuthal dynamics.

### 3.1. Background meridional magnetic field

For the meridional magnetic field, we adopt an analytical model given by Banaszkiewicz et al. (1998). In the present implementation, the model magnetic field consists of the dipole and current-sheet components only. A set of parameters $M = 2.2265, Q = 0, K = 0.9343$ and $a_1 = 1.5$ are chosen such that the last open magnetic field line is anchored at heliocentric colatitude $\theta = 50^\circ$ on the Sun, while at the Earth orbit, the meridional magnetic field strength $B_l$ is $3y$ and independent of colatitude $\theta$, consistent with Ulysses measurements (Smith & Balogh 1995).

The background magnetic field configuration is depicted in Fig. 1, where the thick contours labeled $F$ and $S$ represent the lines of force along which we examine the fast and slow solar wind solutions, respectively. Tube $F-S$, which intersects the Earth orbit at $70^\circ$ (89°) colatitude, originates from $\theta = 38.5^\circ$ (49.4°) at the Sun where the meridional magnetic field strength $B_l$ is $3.93 \times 4.93 \text{ G}$.

### 3.2. Prescribed meridional flow profiles

The background meridional flow parameters are found by adopting a three-step approach described as follows:

1. Using some ad hoc heating parameters, we solve along flux tube $F$ (S) the isotropic version of Eqs. (A.1) to (A.4) (see Eqs. (8) to (10) in Li & Li 2007, for details) to yield the distribution between 1 $R_\odot$ and 1 AU of the ion number densities $n_i$ and meridional speeds $v_{\phi,i}$ ($k = p, \alpha$, as well as the isotropic species temperatures $T_s$ ($s = e, p, \alpha$) for the fast (slow) solar wind. Specifically, the heating rates are of the same format as in Sect. 3.2 in Li & Li (2008). To generate the fast and slow solar wind solutions, the parameters $[F_E$ (in erg cm$^{-2}$ s$^{-1}$), $I_q$ (in $R_\odot$), $\chi$] are chosen to be [1.9, 2.2, 2.2] and [1, 1.8, 3.7], respectively. By simply adjusting the heating parameters it proves difficult to produce a reasonable $T_p$ profile in that $T_p$ in the inner corona is close to observations then $T_p$ at 1 AU is usually only a fraction of the...
typically measured values. Moreover, the derived speed difference $\nu_{op\perp} = \nu_{op} - \nu_{ql}$ varies little between 0.3 and 1 AU, in contrast to the Helios measurements. Therefore some additional steps are employed to make the flow profiles more realistic. Specifically, all the parameters except $n_p$ and $\nu_{ql}$ are required to undergo a smooth transition from the profiles for the region $r \lesssim 0.3$ AU derived so far to those specified in next step for the outer region.

2. The desired profiles, given in Table 1, for $v_{up\perp}, T_p$ and $T_\alpha$ for the region $r \gtrsim 0.3$ AU (denoted by subscript $\alpha$) are based on the in situ measurements to be detailed shortly. Once $v_{up\perp}$ is known, the meridional alpha speed $v_{\alpha\perp}$ is given by $v_{\alpha\perp} + v_{op\perp}$, and the alpha density $n_\alpha$ by $(n_p)(v_{\alpha\perp})/v_{\alpha\parallel}$, where the subscript $I$ denotes the values obtained in the first step. The distributions of $n_\alpha, v_{\alpha\perp}$ ($k = \rho,\alpha$) and $T_\alpha$ ($s = e,\rho,\alpha$) thus constructed are for the isotropic model.

3. Now the ion temperatures $T_s^\perp/k$ can be constructed by prescribing the temperature anisotropy $\Gamma_\alpha = T_s^\perp/k_{s,k}^\perp$ ($k = \rho,\alpha$). Note that the electron temperature is assumed to be isotropic. For the region within several solar radii, $\Gamma_\alpha$ is required to decrease with $r$ from 1 at $1 R_\odot$, where the Coulomb self-collisions are still frequent enough to suppress a temperature anisotropy, to some value less than unity. This inner profile is not directly constrained by observations but constructed by noting that the processes operational in the inner corona tend to heat the ions preferentially in the perpendicular direction (e.g., Hollweg & Isenberg 2002). On the other hand, for $r \gtrsim 0.3$ AU, $\Gamma_\alpha$ follows a power law dependence on $r$ with the exponent determined by the Helios measurements (see Isenberg 1984). Specializing to an electron-proton-alpha plasma, the dispersion relation dictates that instability occurs when $1 - P^-/P^0 > 2(1 - \chi_x/k_\alpha)/\beta^2$ where $\chi_x = (\rho_x/\rho)p_{up\perp}/p_\alpha$ ($k = \rho,\alpha$) with $v_\alpha = B/\sqrt{4\pi\rho}$ being the Alfvén speed determined by the bulk mass density $\rho_\alpha = \rho_\rho + \rho_\alpha$. Using this criterion it is found that the modeled flow profiles are all stable with the only exception being for the segment $r \gtrsim 195 R_\odot$ in the fast wind with the largest values of $\Gamma_{p\parallel}$ and $\Gamma_{\alpha\parallel}$.

Figure 2 gives the radial distributions between 1 $R_\odot$ and 1 AU of the flow parameters for the fast and slow solar wind in the left and right panels, respectively. Figures 2a and c depict the meridional ion speeds $v_{p\parallel}$ and $v_{\alpha\parallel}$, while the ion temperatures $T_s^\parallel/k_{s,k}^\parallel$ follow from the relations $T_s^\parallel = 3T_k/(2 + \Gamma_\alpha)$ and $T_k^\parallel = \Gamma_\alpha T_k^\perp$. A detailed description of Table 1 is necessary. Note that throughout this table $x = r/r_{12}$ where $r_{12} = 1$ AU. Let us focus on the adopted values for the fast solar wind. For the isotropic proton and alpha particles at 1 AU, we adopted the typical values of $T_p = 2.8 \times 10^8 K$ and $T_\alpha = 5T_p$ (see e.g., Schwenn 1990; McComas et al. 2000, hereafter Sch90 and Mc00). Furthermore, Figs. 18 and 19 in Marsch et al. (1982b, hereafter M82b) indicate that $T_k^\perp \propto x^{-0.75}$, and $T_k^\parallel \propto x^{-1.08}$. A power law dependence for $T_p$ of $T_p \propto x^{-1}$ is therefore consistent with such a behavior, and also consistent with the Ulysses measurements (see Table 2 in Mc00). Furthermore, Fig. 5 in Marsch et al. (1982a, hereafter M82a) indicates that $T_k^\perp \propto x^{-1.15}$, and $T_k^\parallel \propto x^{-1.38}$. A profile of $T_\alpha \propto x^{-1.3}$ is consistent with this behavior, but differs substantially from that measured by Ulysses, which yields that $T_\alpha \propto x^{-0.8}$ (see Table 2 in Mc00). Moving on to the slow solar wind, we note that values of $T_p$ are $5.5 \times 10^8 K$ and $T_\alpha = 1.7 \times 10^5 K$ are typically found at 1 AU (see e.g., Sch90). In addition, Figs. 18 and 19 in M82b indicate that $T_p^\perp \propto x^{-1.03}$, and $T_p^\parallel \propto x^{-0.9}$. Therefore we adopted a $T_p$ profile of $T_p \propto x^{-0.94}$. On the other hand, we adopted a profile for $T_\alpha$ in the form $T_\alpha \propto x^{-0.9}$, which is consistent with the measured alpha temperature anisotropies which indicate that $T_\alpha^\perp \propto x^{-0.85}$, and $T_\alpha^\parallel \propto x^{-1.02}$ (see Fig. 5 in M82a).

In this study $\Gamma_{p\parallel}$ and $\Gamma_{\alpha\parallel}$ will serve as free parameters. The Helios measurements indicate that $\Gamma_{p\parallel} \approx 1.2 \pm 0.3$ and $\Gamma_{\alpha\parallel} \approx 1.3 \pm 0.6$ for the fast solar wind with $\nu_{ql} \approx 600$ km s$^{-1}$, while $\Gamma_{p\parallel} \approx 1.7 \pm 0.7$ and $\Gamma_{\alpha\parallel} \approx 1.4 \pm 0.6$ for the slow solar wind with $\nu_{ql} \leq 400$ km s$^{-1}$ (Marsch et al. 1982a,b). Theoretically, one may expect that the $[\Gamma_{p\parallel},\Gamma_{\alpha\parallel}]$ pair may not occupy the whole rectangle bounded by the given values in the $\Gamma_{p\parallel}$-$\Gamma_{\alpha\parallel}$ space, since too strong an anisotropy can drive the system unstable with respect to a number of instabilities when the plasma $\beta$ is comparable to unity. Given that the lower limit of $\Gamma_{p\parallel}$ or $\Gamma_{\alpha\parallel}$ is only slightly lower than 1, the ion-cyclotron instability can be shown to be unlikely to occur (see, e.g., Eq. (3) in Gary et al. 1994). However, the firehose instability may be relevant since it happens when $P^0$ is sufficiently larger than $P_{\alpha\perp}$ and $\beta = 8\pi P^0/B^2 \gtrsim 1$. Note that the alpha particles with a non-negligible abundance drifting relative to protons may complicate the situation considerably given that in addition to the firehose, electromagnetic ion/ion instabilities may also be relevant and the occurrence of such instabilities is not restricted to the cases where the parallel $\beta$ is large (Hellinger & Trávníček 2006). Nevertheless, we only compare the modeled $[\Gamma_{p\parallel},\Gamma_{\alpha\parallel}]$ with the non-resonant firehose criterion such as found via the dispersion relation of Alfvén waves (see Eq. (23) in Isenberg 1984). Specializing to an electron-proton-alpha plasma, the dispersion relation dictates that instability occurs when $1 - P^-/P^0 > 2(1 - \chi_x/k_\alpha)/\beta^2$ being the Alfvén speed determined by the bulk mass density $\rho_\alpha = \rho_\rho + \rho_\alpha$. Using this criterion it is found that the modeled flow profiles are all stable with the only exception being for the segment $r \gtrsim 195 R_\odot$ in the fast wind with the largest values of $\Gamma_{p\parallel}$ and $\Gamma_{\alpha\parallel}$.

For the fast (slow) solar wind it is found that at 1 AU the meridional proton speed $\nu_{p\parallel} = 607$ (304) km s$^{-1}$, the proton flux $n_p\nu_{p\parallel} = 2.8$ (3.84) in units of $10^6$ cm$^{-2}$ s$^{-1}$, the alpha abundance $n_\alpha/n_p = 4.56\%$ (3.6$\%$), and the meridional component of the proton-alpha velocity difference $\nu_{op\perp}$ is 23 (5) km s$^{-1}$. These values are consistent with in situ measurements such as made by Ulysses (McComas et al. 2000). Moreover, the fast (slow) solar wind reaches the Alfvén point at 10.7 (13.3) $R_\odot$, beyond which $\nu_{ql}$ increases only slightly with increasing $r$. On the other hand, for $r \gtrsim 0.3$ AU the meridional alpha speed $\nu_{\alpha\parallel}$ decreases rather than increases with $r$ as a consequence of the prescribed $\nu_{op\perp}$ profile. If examining the ratio of $\nu_{op\perp}/\nu_{op\parallel}$ one may find that for the fast solar wind this ratio decreases only slightly from 0.98 at 0.3 AU to 0.82 at 1 AU, while for the slow wind it shows a substantial variation from 0.88 at 0.3 AU to 0.29 at 1 AU. The modeled $\nu_{op\perp}/\nu_{op\parallel}$ can be seen to agree with the Helios measurements as given by Fig. 11 of Marsch et al. (1982a). Note that a value of $\nu_{op\perp} = 49$ km s$^{-1}$ at 0.3 AU is not unrealistic for slow solar winds, even larger values have been found by Helios 2 when approaching perihelion (Marsch et al. 1981). Moving on to the temperature profiles, one may see that the $T_k^\perp$ profiles inside 5 $R_\odot$ are in reasonable agreement with the determined values there. Finally, $v_{\alpha\perp}$ decreases with $r$ in the slow wind. For the fast solar wind, $v_{\alpha\perp}$ is larger and decreases with $r$. The slow wind may not be slow enough to keep the $v_{\alpha\perp}$ value constant, but the $v_{\alpha\perp}$ value increases when $r$ increases.
and Tk and a) et al. (2003) for a streamer, respectively. Note that both measurements are typical of solar minimum conditions. Moreover, the asterisks in denote the Alfvén point, where the meridional Alfvénic Mach number (defined by Eq. (18)) equals unity.

c) speed vnegative values. In Figs. 3a and b, the ion azimuthal speeds for A

to this end, let us first examine the fast and then the slow so-

c) momentum budget distributed among particle momenta, the mag-

4. Numerical results

Having described the meridional magnetic field and flow pro-

c) fast and slow solar wind.

4.1. Fast solar wind

Figure 3 presents the radial profiles of (a) the proton azimuthal speed vφp; (b) the alpha one vφα; and (c) the ion angular momentum fluxes Lk (k = p, α), their sum LP, the flux due to the magnetic torque LM, and that due to temperature anisotropies Lani (see Eq. (16)). Note that the dash-dotted curves in Fig. 3c plot negative values. In Figs. 3a and b, the ion azimuthal speeds for the isotropic model with identical meridional flow parameters are given by the dashed lines for comparison. The fast wind profile corresponds to ΓpE = 1.5 and ΓαE = 1.9.

For the chosen ΓpE and ΓαE, it is found that λ = 1.058. Consequently, the total angular momentum loss rate per steradian L is 1.8 (here and hereafter in units of 1020 dyne cm s⁻¹) in the anisotropic case, and is only modestly enhanced compared with the isotropic case, for which L = 1.71. Furthermore, Figs. 3a and b indicate that the radial dependence of the ion azimuthal speed vφp or vφα in the anisotropic model is similar to that in the isotropic one. For instance, both models yield that with increasing distance the alpha particles develop an azimuthal speed in the direction of counterrotation with the Sun: vφα becomes negative beyond 7.95 (8.35) R⊙ in the anisotropic (isotropic) model. The difference between the isotropic and anisotropic cases becomes more prominent at large distances where βp becomes increasingly significant, as would be expected from Eq. (17). Take the values of vφp and vφα at 1 AU. The isotropic (anisotropic) model yields that vφp = 2.54 (3.46) km s⁻¹ and that vφα = −12.6 (−11.7) km s⁻¹ at 1 AU. Note that the changes introduced to the ion azimuthal speeds by pressure anisotropies (−0.9 km s⁻¹ for both protons and alpha particles) play an important role in the distribution of the angular momentum budget L among different contributions, as shown by Fig. 3c. The proton contribution Lp exceeds Lα for r ≥ 57 R⊙ and Lp attains 5.07 at 1 AU, significantly larger than the magnetic part LM = 1.48 at the same location. In fact, the overall particle contribution LP, which increases with distance, overtakes the magnetic contribution Lm from 170.5 R⊙ onwards, despite the fact that the alpha contribution tends to offset the proton one. The dominance of Lp over Lα happens in conjunction with the increasing importance of Lα, the flux due to total pressure anisotropy which is in the direction of counterrotation with the Sun. In contrast, without pressure anisotropies, at 1 AU it turns out that even though a value of 3.73 is found for Lp, it is almost cancelled by an Lα of −3.49. The resulting LP is thus 0.23, substantially smaller than agreement with the UVCS line-width measurements for both the fast and slow solar wind.

Fig. 2. Radial distribution between 1 R⊙ and 1 AU of the adopted meridional flow parameters for the fast (left column) and slow (right) solar wind. a) and c), the meridional flow speeds of protons (vφp) and alpha particles (vφα). b) and d), the ion temperatures Tp (dotted lines), Tα (solid lines) and Tk (dashed lines), and Tl = (T∥ + 2T⊥)/3 (solid lines) where k = p, α. The construction of Tp∥ is described in Sect. 3.2. The error bars in b) and d) represent the uncertainties of the UVCS measurements of the effective proton temperature as reported by Kohl et al. (1998) for a coronal hole, and by Frazin et al. (2003) for a streamer, respectively. Note that both measurements are typical of solar minimum conditions. Moreover, the asterisks in a) and c) denote the Alfvén point, where the meridional Alfvénic Mach number (defined by Eq. (18)) equals unity.
Table 1. Profiles for some solar wind parameters in the region \( r > 0.3 \text{ AU} \).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fast wind</th>
<th>Slow wind</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (v_{\alpha \phi})_p )</td>
<td>( 23 \times 10^{15} \text{ km s}^{-1} )</td>
<td>( 5 \times 10^{15} \text{ km s}^{-1} )</td>
</tr>
<tr>
<td>( (T_{\alpha \phi})_p )</td>
<td>2.8 \times 10^7 \text{ K}</td>
<td>5.5 \times 10^7 \text{ K}</td>
</tr>
<tr>
<td>( (T_{\alpha \phi})_p )</td>
<td>1.4 \times 10^8 \text{ K}</td>
<td>1.7 \times 10^8 \text{ K}</td>
</tr>
<tr>
<td>( \Gamma_{\phi \alpha \phi} )</td>
<td>( \Gamma_{\phi \alpha \phi}^{0.035} )</td>
<td>( \Gamma_{\phi \alpha \phi}^{0.013} )</td>
</tr>
<tr>
<td>( \Gamma_{\phi \alpha \phi} )</td>
<td>( \Gamma_{\phi \alpha \phi}^{0.23} )</td>
<td>( \Gamma_{\phi \alpha \phi}^{0.19} )</td>
</tr>
</tbody>
</table>

* Please see Sect. 3.2 for details.

\( L_M \), which is nearly identical to the value found in the anisotropic model. This contrast between anisotropic and isotropic cases is understandable since it follows from Eq. (13) that, given that the constant \( \Lambda_L \) is constant under the same set of conditions, the overall angular momentum flux \( L_p \) should be largely offset by that of \( L_{\alpha \phi} \).

Figure 4 expands the obtained results by displaying the dependence of \( \Gamma_{\phi \alpha \phi} \) and \( \Gamma_{\phi \alpha \phi} \) on the factors \( \alpha \) and \( \phi \). The arrows have been slightly shifted from one another to avoid overlapping.

From Fig. 4a one can see that \( \alpha \) decreases with increasing \( \Gamma_{\phi \alpha \phi} \), ranging from 1.101 at the upper left to 1.058 at the lower right corner. The deviation of \( \alpha \) from unity, albeit modest, indicates that the changes introduced in the total angular momentum loss due to the ion pressure anisotropies are not negligible. From Figs. 4b and c one can see that between 0.3 to 1 AU, the magnitude of the azimuthal speed \( v_{\alpha \phi} \) at both distances decreases with increasing distance. Furthermore, at either 0.3 or 1 AU, both \( v_{\alpha \phi} \) and \( v_{\phi \alpha} \) increase when \( \Gamma_{\phi \alpha \phi} \) or \( \Gamma_{\phi \alpha \phi} \) increases. The values at 1 AU for each parameter can be obtained from the relationship between the values at any given \( \Gamma_{\phi \alpha \phi} \) and the magnetic one \( L_M \), the overall particle contribution \( L_p = L_p + L_M \) is also given. Note that \( -L_p \) instead of \( L_p \) is plotted in Fig. 4d. Moreover, the horizontal bars on the left of Figs. 4b and c represent the azimuthal ion speeds derived in the isotropic case at the corresponding locations for comparison. The open circles correspond to the cases where \( \Gamma_{\phi \alpha \phi} = 1.3 \). It turns out that at any given \( \Gamma_{\phi \alpha \phi} \) each parameter varies monotonically from the value with \( \Gamma_{\phi \alpha \phi} = 0.7 \), represented by the end of the arrow, to the value with \( \Gamma_{\phi \alpha \phi} = 1.9 \) given by the arrow head. In Fig. 4b the arrows have been slightly shifted from one another to avoid overlapping.

4.2. Slow solar wind

Figure 5 presents, in the same fashion as Fig. 4, the dependence of \( \Gamma_{\phi \alpha \phi} \) and \( \Gamma_{\phi \alpha \phi} \) on \( \Gamma_{\phi \alpha \phi} \) and \( \Gamma_{\phi \alpha \phi} \) of various quantities derived for the slow solar wind. A comparison with Fig. 4 indicates that nearly all the features in Fig. 5 are reminiscent of those obtained for fast solar wind.
solar wind solutions. However, some quantitative differences exist nonetheless. For instance, when $\Gamma_{pe}$ is held fixed, all the examined parameters for the slow wind vary little even though $\Gamma_{pe}$ changes considerably from 0.8 to 2. In contrast, the parameters for the fast wind show an obvious $\Gamma_{pe}$ dependence. This difference can be largely attributed to the fact that in the slow wind the ions are substantially cooler than in the fast wind. Figure 5a shows that $\lambda$ ranges from 0.94 to 1.016. In other words, relative to the isotropic case, the solar angular momentum loss rate per steradian in the anisotropic models may be enhanced or reduced by up to 6%. If examining Figs. 5b and c, one may find that at both 0.3 and 1 AU, the azimuthal speeds of both ion species, $v_{\phi,p}$ and $v_{\phi,e}$, are larger algebraically in the anisotropic models than in the isotropic one. The difference between the two is more prominent at 0.3 AU, where the isotropic model yields $[v_{\phi,p}, v_{\phi,e}] = [3.49, -18.1] \text{ km s}^{-1}$, whereas the anisotropic models yield that with increasing $\Gamma_{pe}$, $v_{\phi,p}$ increases from 3.76 to 5.04 km s$^{-1}$, and $v_{\phi,e}$ varies between $-17.8$ to $-163 \text{ km s}^{-1}$. As for the ion azimuthal speeds at 1 AU, one can see that varying $\Gamma_{pe}$ leads to a $v_{\phi,p}$ varying between 1.18 and 1.72 km s$^{-1}$, and a $v_{\phi,e}$ ranging from $-5.85$ to $-5.31 \text{ km s}^{-1}$. The corresponding changes in the specific ion angular momentum fluxes are shown by Fig. 5d, which indicates that the proton one $L_p$ increases with increasing $\Gamma_{pe}$ from 2.52 to 3.67, and likewise, the alpha one $L_\alpha$ increases from $-1.83$ to $-1.66$. On the other hand, the flux associated with magnetic stresses $L_M$ hardly varies, and a value of 3.36 can be quoted for all the models examined. Therefore in the parameter space explored, $L_M$ may be smaller than $L_p$, which is however offset by the alpha contribution that is always in the direction of counter-rotation to the Sun. In fact, the alpha contribution is so significant that the overall particle contribution $L_p$
results in the isotropic model. To construct Fig. 6, all the possible 
values of $\Gamma_{\mathrm{E}}$ and $\Gamma_{\mathrm{AE}}$ have been examined. As a result, at any 
radial location the ratio $\delta u_\alpha/\delta u_\mathrm{p}$ varies from model to model, 
and the range in which this ratio may occupy is given by the 
hatched area. The zero-frequency solutions are obtained by solv-
ing Eq. (20), while for hydromagnetic WKB Alfvén waves it is 
well known that $\delta u_\alpha/\delta u_\mathrm{p} = (\epsilon_\mathrm{p} - \epsilon_\alpha)/\epsilon_\mathrm{p}$, where $\epsilon_\mathrm{p}$ is the 
wave phase speed and given by (e.g., Barnes & Suffolk 1971; 
Isenberg 1984)

\[ v_\mathrm{ph} = v_\mathrm{cm} + \sqrt{4\pi \alpha^2 \left(1 - \frac{\pi P_{\delta S}}{B^2}\right)} - \rho_\alpha \rho_\mathrm{p} v_\alpha^2, \]

in which $v_\mathrm{cm}$ is the center of mass velocity, and $\rho_\alpha/\rho_\mathrm{p}$ defines the fractional ion mass density.

From Fig. 6 one can see that the zero-frequency and 
WKB solutions are well separated from each other, in the 
isotropic and anisotropic cases alike. For the isotropic model, 
$\delta u_\alpha/\delta u_\mathrm{p}$ in the zero-frequency case increases monotonically from 
3.79 at 40 $R_\odot$ to 4.68 at 100 $R_\odot$. On the other hand, in 
the WKB case it decreases first from 0.22 at 40 $R_\odot$ and attains 
its minimum of 0.083 at 60.3 $R_\odot$ and then increases to 0.15 at 
100 $R_\odot$. The difference in $\delta u_\alpha/\delta u_\mathrm{p}$ between the zero-frequency 
and WKB solutions may be slightly smaller in the anisotropic 
but the difference is still quite significant. From this we can 
conclude that, with realistic ion temperature anisotropies included, 
the alpha velocity fluctuation spectrum $S_\alpha(f)$ during Alfvénic 
activities will also show an apparent break near $f_c$, if the proton 
one $S_\mathrm{p}(f)$ is smooth there. This break is entirely a linear prop-
erty, and has nothing to do with the nonlinearities that may also 
shape the fluctuation spectra.

6. Summary

This study has been motivated by the apparent lack of an analysis 
on the angular momentum transport in a multicomponent solar 
or stellar wind with differentially flowing ions and species 
temperature anisotropy. Moreover, there has been an outstanding 
discrepancy between available measurements and models 
concerning the relative importance of the particle $L_\mathrm{p}$ and mag-
netic contribution $L_\mathrm{M}$ to the solar angular momentum loss rate 
per steradian $L$. The Helios measurements indicate that for fast 
(slow) solar wind with $\eta_\mathrm{p} \simeq 600$ ($\leq 500$) km s$^{-1}$, $L_\mathrm{p}$ tends to be 
positive (negative), with the positive sign denoting the direction 
of corotation with the Sun. Furthermore, $L_\mathrm{p}$ tends to be larger 
than $L_\mathrm{M}$ in the slow wind. The behavior of $L_\mathrm{p}$ derives from that of 
individual ion angular momentum fluxes, $L_\alpha$ and $L_\mathrm{p}$, thereby 
calling for a multifluid approach.

Starting with a general set of multifluid transport equations 
with gyrotropic species pressure tensors, we have derived the equations for both the angular momentum conservation 
(Eqs. (10) and (20) in Sect. 2), and the energy and linear moment-
um balance (Eqs. (A.1) to (A.4) in the Appendix). These equations 
are not restricted to radial outflows in the equatorial plane, 
instead they are valid for arbitrary axisymmetrical winds that in-
clude two major ion species, and therefore are expected to find 
applications in general outflows from late-type stars. To focus 
on the problem of angular momentum transport, we refrained 
from solving the full set of equations governing the meridional 
dynamics. Rather, we constructed, largely based on the avail-
able in situ measurements, the empirical profiles for the merid-
ional magnetic field and flow parameters. Only the ion tempera-
ture anisotropies are considered, i.e., the electron temperature is

Fig. 6. Radial dependence of the ratio of the alpha to the proton ve-
locity fluctuation amplitudes $\delta u_\alpha/\delta u_\mathrm{p}$ induced by Alfvénic activities in 
super-Alfvénic portions of the fast solar wind. The dashed curves corre-
respond to the isotropic model, while the hatched areas give the possible 
range $\delta u_\alpha/\delta u_\mathrm{p}$ may occupy when the parameters $\Gamma_{\mathrm{E}}$ and $\Gamma_{\mathrm{AE}}$ vary in the 
ranges given in text. Both the zero-frequency (upper portion) and WKB 
(lower portion) estimates are given.

never exceeds $L_\mathrm{M}$. In other words, incorporating ion temperature 
anisotropy cannot resolve the outstanding discrepancy between 
previous models and observations concerning the relative im-
portance of particle and magnetic contributions in the angular 
momentum budget of the solar wind.

5. Discussion

As demonstrated by Li & Li (2008), the discussion on the an-
gular momentum transport also allows us to say a few words on the 
frequency spectra $S_\alpha(f)$ ($k = p, \alpha$) of the ion velocity 
fluctuations during Alfvénic activities in the fast solar wind in the 
super-Alfvénic portion where $M_\odot^2 > 1$. This is due to the 
well-known change of the properties of Alfvénic fluctuations 
around some $f_c \approx v_\mathrm{cm}/(4\pi r_\odot)$, where $v_\mathrm{cm}$ is the speed of center 
of mass evaluated at the Alfvén point $r_\odot$ (see e.g., Heinemann 
& Albert 1980; Li & Li 2008). For typical fast wind parameters, 
$f_c \approx 0.5 - 1 \times 10^{-5}$ s$^{-1}$. While the fluctuations with fre-
cuencies $f \lesssim f_c$ are genuinely wave-like and may be described 
by the WKB limit given the slow spatial variation of flow param-
eters in the region in question, those with $f \gtrsim f_c$ behave in a 
 quasi-static manner and may be described by the solutions 
to the angular momentum conservation law which also governs 
the zero-frequency fluctuations. As shown by Li & Li (2008) 
who neglected the species temperature anisotropy, in the region 
r \gtrsim 0.2 AU which will be explored by the Solar Orbiter and Solar 
Probe, the ratio of the alpha to proton velocity fluctuation am-
plitude $\delta u_\alpha/\delta u_\mathrm{p}$ can be an order-of-magnitude larger for $f < f_c$ 
than for $f > f_c$. Hence one may expect that, if the proton velocity 
fluctuation spectrum $S_\alpha(f)$ is somehow smooth around $f_c$, then 
the alpha one $S_\alpha(f)$ will show an apparent spectral break. Now 
let us revisit this problem in light of the discussion presented in 
this paper and see what changes the pressure anisotropies may 
introduce.

Restrict ourselves to either the high-latitude region or the 
region inside say 100 $R_\odot$ such that the magnetic field may be 
seen as radial. Furthermore, suppose that the waves are propa-
gating parallel to the magnetic field in the empirical fast wind 
profiles detailed in Sect. 3. Figure 6 presents the radial depen-
dence of $\delta u_\alpha/\delta u_\mathrm{p}$ in the region between 40 and 100 $R_\odot$ for both 
the zero-frequency (upper part) and WKB (lower part) solutions. 
For comparison, the dashed curves represent the corresponding
seen as isotropic. For both the fast and slow solar wind profiles, we solved the angular momentum conservation law (Eqs. (10) and (20)) to examine how the azimuthal speeds of protons $v_{p\phi}$ and alpha particles $v_{\alpha\phi}$, as well as the individual components in the solar angular momentum budget are influenced by the ion temperature anisotropies. To this end, solutions to the isotropic version are obtained for comparison.

Our main conclusions are:

1. From the derived equations governing the energy transport, a simple analysis given in the Appendix yields that the adiabatic cooling may be considerably influenced with the introduction of the azimuthal components. Such an influence is understandably more prominent in the low-latitude regions. This means, when modeling the species temperature anisotropy, for a quantitative comparison of model computations to be made with the near-ecliptic measurements such as made by Helios, the spiral magnetic field has to be taken into account.

2. In agreement with the single-fluid case (Weber & Davis 1970; Weber 1970), incorporating species temperature anisotropy leads to a situation where the total angular momentum loss rate per steradian $L$ is determined by the behavior of the solution to the angular momentum conservation law in the vicinity of the Alfvén point where the combined Alfvénic Mach number $M_F = 1$. However, $M_F$ has to be taken into account the contribution from both ion species, as defined by Eq. (18).

3. Relative to the isotropic case, the introduced species temperature anisotropy may enhance or decrease $L$ by up to 10%, and introduce an absolute change of up to $\sim 1.8$ km s$^{-1}$ in individual ion azimuthal speeds in the region between 0.3 and 1 AU. While these changes seem modest, the corresponding changes in the angular momentum fluxes convected by protons $L_p$ or alpha particles $L_{\alpha}$ may change substantially. In contrast, the flux associated with magnetic stresses $L_M$ hardly varies.

4. However, introducing ion temperature anisotropies cannot resolve the discrepancy between in situ measurements and models. For the fast wind solutions, while in extreme cases $L_p$ may become negative $L_\alpha$ always stays positive. On the other hand, for the slow solar wind solutions examined, $L_p$ never exceeds $L_M$ even though $L_M$ may be smaller than the individual ion contribution. This is because, for both the slow and fast wind solutions, $L_p$ and $L_\alpha$ always have opposite signs.

5. The discussion on the angular momentum transport has some bearing on the ion velocity fluctuation spectra $S_{\phi}(f)$ $(k=p, \alpha)$ during Alfvénic activities in the super-Alfvénic regions, which are likely to be explored by future missions such as Solar Orbiter and Solar Probe. In agreement with Li & Li (2008) where species temperature anisotropies are neglected, an analysis based on the WKB and zero-frequency solutions yields that $S_{\phi}(f)$ will show an apparent break around some critical frequency $f_c$ if $S_{\phi}(f)$ is smooth there. This $f_c \sim 0.5-1 \times 10^3$ s$^{-1}$ is the well-known frequency that separates the genuinely wave-like fluctuations from quasi-static ones.

Appendix A: Derivation of equations governing the meridional dynamics

In Sec. 2, we have demonstrated that the vector equations governing a time-independent multicomponent solar wind with species temperature anisotropy are allowed to be decomposed into a force balance condition across the poloidal magnetic field and a set of transport equations along it. The azimuthal dynamics has been discussed in the text, whereas this Appendix provides some discussion on the poloidal dynamics. In particular, we shall derive the equations governing the poloidal motion $v_{l\phi}$ of ion species $(k=p, \alpha)$, and the species temperatures $T_{k\phi}^{\perp} (s = e, p, \alpha)$ in rather general situations.

Due to the presence of $v_{lN}$ in the $l$ component of the ion momentum Eq. (2), one may expect that the $N$-component of Eq. (2) has to be solved. In fact, there is no need to do so because $v_{lN}$ appears only in the difference $v_{lN} - v_{lN}$, which may be found from the $\phi$ component of Eq. (2). Substituting $v_{lN} - v_{lN}$ into the $l$ component of Eq. (2) will then eliminate the cumbersome $\Omega_l$ and $\Omega_{lN}$. Note that this technique, first devised by McKenzie et al. (1979), ensures the conservation of not only total momentum but also total energy (see Li & Li 2006). Specifically, the resulting equations for the poloidal dynamics are

\[ \frac{1}{B_l} \left( \frac{\partial v_{l\phi}}{\partial t} \right)' = 0, \quad (A.1) \]

\[ v_{l\phi}(R)' + \frac{\phi}{R} \frac{\partial}{\partial R} \left( \frac{\phi}{R} \right)' - (C_l + \frac{\phi \Omega_l}{R})' = 0, \quad (A.2) \]

\[ \frac{1}{n_k B_l} \left\{ \nabla \cdot \frac{\partial}{\partial R} \left( c_{te} \frac{\partial}{\partial R} \right) + \frac{\partial E_{B\phi}}{\partial t} + \frac{H_{B\phi}}{R^2} \right\} = 0, \quad (A.3) \]

\[ v_{l\phi} \left( T_{\phi l}^{\perp} \right)' - v_{l\phi} T_{\phi l}^{\perp} \left[ \ln (v_{l\phi} \sec \phi) \right]' = 0, \quad (A.4) \]

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that \((\ln B)_{y'} = -2/r\). On the other hand, using the alignment condition (10) one may find that
\[
v_{k}(v_{k})' - v_{k}R_{f}(\ln R)' + \tan \Phi \frac{v_{k}u_{l}}{R}(R_{kl})' = \left(\frac{v_{k}^{2}}{2} \sec^{2} \Phi \right)' - \left(\frac{A^{2}R^{3}}{2}\right)'. \tag{A.5}\]
Again specializing to a spherical solar wind, one then finds that Eq. (A.2) is equivalent to (A.2) in Isenberg (1984). It should be noted that although working in frame of reference corotating with the Sun, as did Isenberg (1984), substantially simplifies the algebra, it does not offer the information on the specific form of the spiral angle \(\Phi\), whose functional dependence on the flow speeds has to be assumed a priori. In practice, Isenberg (1984) assumed that the velocity of center of mass \(v_{cm}\) is radial in an inertial frame beyond \(10R_{\odot}\), which is certainly a good assumption for the present slow-rotating Sun. However, from our discussion on the azimuthal components, there is in general no guarantee that \(v_{cm}\) is radial, and the deviation may be substantial for winds that flow from a faster rotating star.

Introducing azimuthal components may influence the ion flow speeds \(v_{kl}\) both directly and indirectly. The direct consequence is that azimuthal components may introduce into the reduced meridional momentum Eq. (A.2) an effective force (see the first three terms). Note that in a corotating frame the magnitude of the ion velocity becomes \(v_{kl}\ sec \Phi\) from relation (A.5) one may see that in such a frame all particles move in the same centrifugal potential \(A_{2}R^{2}/2\). Therefore in effect the introduced force tends to reduce the magnitude of the ion speed difference with increasing distance as \(sec \Phi\) tends to increase. This effect has been explored in detail in Li & Li (2006) and Li et al. (2007), where it is shown that the influence may play an important part in the force balance for the solar wind. In fact, introducing solar rotation alone is able to reproduce the \(v_{pp}\) profile measured by Ulysses beyond 2 AU if a proper value of \(v_{pp}\) is imposed there. On the other hand, \(v_{kl}\) may be altered indirectly by the modified pressure gradient force due to changes in the temperatures, which in turn are caused by the changes in the heat fluxes (the third term in Eqs. (A.3) and (A.4)) and through the adiabatic cooling (the second term). A detailed discussion on the former requires a specific form for the heat flux, which is beyond the scope of the present paper. As a consequence, we shall focus on the latter instead.

Neglecting the terms in the second pair of square parenthesis, Eqs. (A.3) and (A.4) give
\[T_{i}^{j}o = \cos^{2} \Phi / v_{k}R_{kl}, T_{i}^{j}o = B_{l} sec \Phi. \tag{A.6}\]
Note that the relation governing \(T_{i}^{j}o\) simply reflects the conservation of magnetic moment. Now that in the region say \(r > 10 R_{\odot}\ sec \Phi\) is significant and increases with \(r, T_{i}^{j}o (T_{o}^{j})\) may be substantially reduced (enhanced) relative to the case where \(\Phi \equiv 0\). This effect is particularly significant in the near-ecliptic region and for the slow solar wind. For instance, restrict ourselves to the equatorial plane and consider the region between say 10 \(R_{\odot}\) and 1 AU. Suppose \(v_{df}\) remains constant and \(v_{df} = A_{0}R_{\odot} = 430 \text{ Km s}^{-1}\). Now that roughly speaking \(\tan \Phi \equiv -A_{0}R_{l}/v_{df}\), when the spiral field is considered, \(T_{o}^{l} (T_{o}^{l})\) at 1 AU is \(1/2 (\sqrt{2})\) times the value for a purely radial magnetic field. This suggests that for making any quantitative comparison of the modeled species temperature anisotropy with the near-ecliptic measurements such as made by Helios, the spiral magnetic field has to be considered.

For completeness, we note that the force balance condition across the \(N\) direction comes from the \(N\) component of the total momentum, which reads
\[
\sum_{k} \rho_{k} \left(v_{k}^{2} \frac{\partial}{\partial N} \ln R + \frac{\partial}{\partial N} \left( p_{k}^{l} + B_{k}^{2} \right) \right) - \frac{1}{4\pi} \left( 1 - \frac{4\pi B_{k}^{2}}{B^{2}} \right) \left( \frac{B_{k}^{2}}{R_{c}} - \frac{B_{k}^{2}}{R_{c}^{2}} \frac{\partial \ln R}{\partial N} \right) + \sum \rho_{k} G_{l} \frac{\partial}{\partial N} \ln r = 0, \tag{A.7}\]
where \(R_{c} = e_{N} \cdot (\hat{e}_{i} \cdot \nabla e_{i})\) is the signed curvature radius of the poloidal magnetic line of force. Obviously, this force balance condition determines the poloidal magnetic field configuration in response to the electric currents associated with the flow. This equation, combined with the transport equations along the meridional magnetic lines of force, may be solved alternately to find a self-consistent solution to the vector equations by using the approach by Pneuman & Kopp (1971) or Sakurai (1985).

References
McComas, D. J., Barralough, B. L., Funsten, H. O., et al. 2000, J. Geophys. Res., 105, 10419 (Mc00)
Schwenn, R. 1990, Large-scale structure of the interplanetary medium, in Physics of the Inner Heliosphere I, Large Scale Phenomena, ed. R. Schwenn, & E. Marsch (New York: Springer-Verlag), 99 (Sch90)