

## Standing Shocks in the Inner Slow Solar Wind \*

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We examine whether the flow tube along the edge of a coronal streamer supports standing shocks in the inner slow wind by solving an isothermal wind model in terms of the Lambert  $W$  function. It is shown that solutions with standing shocks do exist and they exist in a broad area in the parameter space characterizing the wind temperature and flow tube. In particular, streamers with cusps located at a heliocentric distance  $\gtrsim 3.2R_\odot$  can readily support discontinuous slow winds with temperatures barely higher than 1 MK.

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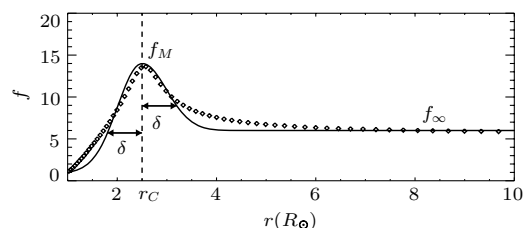
It was proven that the quasi-steady solar wind may not be continuous but involve standing shocks in the near-Sun region.<sup>[1-7]</sup> As first pointed out 30 years ago,<sup>[1]</sup> existence of standing shocks depends critically on existence of multiple critical points (CPs). These can arise due to either momentum addition or rapid tube expansion near the base. Time-dependent simulations have shown that whether or not the system adopts a discontinuous solution depends on the detailed manner by which the tube geometry is varied,<sup>[2,5]</sup> or how the momentum addition is applied.<sup>[4,5]</sup> Existing studies on standing shocks were exclusively on the flow rooted in the interior of coronal holes. However, little is known about whether the flow tubes bordering bright streamer helmets can support standing shocks as well. This region is important since the slow wind likely originates in the region.<sup>[8]</sup> Here the tube expansion is distinct from the coronal-hole one, with the tube likely to experience a dramatic expansion around the streamer cusp (see Fig. 4, the current-sheet case in Ref. [9]). In this Letter, we are interested in answering: Are standing shocks allowed by this geometry?

To isolate the geometrical effect, we use a simple isothermal model. Let  $T$  and  $v_r$  denote the solar wind temperature and radial speed, respectively. The isothermal sound speed is then  $c_s = \sqrt{2k_B T/m_p}$ , where  $k_B$  is the Boltzmann constant and  $m_p$  the proton mass. The Mach number  $M = v_r/c_s$  is governed by<sup>[12]</sup>

$$\left(M - \frac{1}{M}\right) \frac{dM}{dy} = \frac{d \ln \bar{a}}{dy} - \frac{\Delta}{y^2}, \quad (1)$$

where  $y = r/R_\odot$  with  $R_\odot$  being the solar radius and  $r$  the heliocentric distance. Moreover,  $\bar{a} = a/R_\odot^2$  is the non-dimensionalized tube cross-section  $a$ ;  $a$  is related

to the expansion factor  $f$  by  $a(r) = f(r)r^2$ . Furthermore,  $\Delta = g_\odot R_\odot/c_s^2$ , where  $g_\odot$  is the surface gravitational acceleration. Evidently  $\Delta$  measures the relative importance of the gravitational force and pressure gradient force.



**Fig. 1.** Expansion factor  $f$  for the streamer geometry versus heliocentric distance  $r$ . Please see text for the meaning of  $f_\infty$ ,  $f_M$ ,  $r_C$  and  $\delta$  and what the diamonds refer to.

The streamer geometry is parameterized as

$$f(r) = \begin{cases} 1 + (f_M - 1) \frac{G(r; r_C, \delta) - G(R_\odot; r_C, \delta)}{1 - G(R_\odot; r_C, \delta)}, & r \leq r_C, \\ f_\infty + (f_M - f_\infty) G(r; r_C, \delta), & r \geq r_C, \end{cases} \quad (2)$$

where  $G(x; x_0, \delta) = \exp[-(x - x_0)^2/\delta^2]$  is a Gaussian. Figure 1 illustrates the  $r$ -distribution of  $f$ . Obviously  $f_\infty$  represents the value at large distances and  $f_M$  is the maximum attained at  $r_C$ , the heliocentric distance of the streamer cusp. Moreover,  $\delta$  describes how rapid  $f_M$  is approached. For  $r_C$ , we adopt values between 2.4 and  $3.6R_\odot$ , compatible with LASCO C2 images. The ranges for  $f_\infty$ ,  $f_M$  and  $\delta$  are  $[2, 10]$ ,  $[6, 22]$  and  $[0.4, 1]R_\odot$ , respectively. As direct measurements of the coronal magnetic field remain largely unavailable, a model field is used to guide our choice. The  $f$  profile with the base values ( $f_\infty = 6$ ,  $f_M = 14$  and

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$\delta = 0.7R_\odot$ ) is close to the diamonds in Fig. 1, which correspond to  $f$  along the tube at the streamer edge in a current sheet model, given in Fig. 4(b) of Ref. [9] (the one labeled  $27^\circ$ ).

Given the temperature  $T$  and an  $f(r)$ , the right hand side (RHS) of Eq. (1) can be readily evaluated and it determines whether solutions with standing shocks are allowed. To explain this, we note that any root of  $\text{RHS} = 0$  corresponds to a critical point (CP), which is either a local extreme ( $dM/dy = 0, M \neq 1$ ) or a sonic point (SP) ( $dM/dy \neq 0, M = 1$ , denoted by the subscript  $S$ ). Shock solutions are known to exist only when there are multiple CPs, and were usually constructed by carefully examining the solution topology. Here we present a new method based on a recent study which showed that a transonic solution to Eq. (1) is expressible in terms of the Lambert  $W$  function  $W(x)$  [10]

$$M^2 = \begin{cases} -W_0(-D(y)), & 1 \leq y \leq y_S, \\ -W_{-1}(-D(y)), & y \geq y_S, \end{cases} \quad (3)$$

where

$$D(y) = \frac{\bar{a}_S^2}{\bar{a}^2} \exp \left[ 2\Delta \left( \frac{1}{y_S} - \frac{1}{y} \right) - 1 \right]. \quad (4)$$

Only two things about  $W(x)$  need to be known in the present context: first, a real-valued  $W(x)$  can be defined only for  $x \geq -1/e$  (note that  $D$  is positive definite); second,  $W(x)$  has two branches for  $-1/e < x < 0$  and they obey  $-1 \leq W_0 < 0$  and  $W_{-1} \leq -1$ . The mathematical details can be found in Ref. [11]. In practice, we evaluate  $W_0$  and  $W_{-1}$  via Eq. (5.9) there.

If only one CP exists, it is natural the SP and Eq. (3) describes the only possible transonic solution for which  $M$  increases monotonically with  $r$ . This is the case considered in Ref. [10], where  $f \equiv 1$  is assumed. In our case there exist up to 3 CPs and hence we must extend the Lambert  $W$  function approach as follows. First, when 3 CPs exist, only the innermost and outermost ones turn out to be relevant. We evaluate  $D$  by choosing each of them, one after another, as the SP. In some portion of the computational domain ( $y \geq 1$ ),  $D$  for one CP may exceed  $1/e$ , and hence the solution is not defined. Call this solution the ‘‘broken solution’’, denoted by  $M_b$ . Choosing the other CP as SP results in a continuous solution, denoted by  $M_c$ . If standing shocks exist, they must appear at the two positions where the Rankine-Hugoniot relations and evolutionary condition are met. In the isothermal case these translate into [4,12]

$$M^+ M^- = 1 \quad \text{and} \quad M^+ > 1, \quad (5)$$

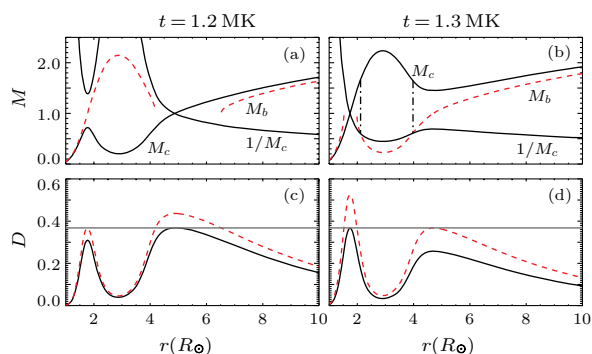
where  $+$  ( $-$ ) represents the shock upstream (downstream). This suggests a simple graphical means to construct solutions with shocks. [12] We plot  $M_b$  and examine whether it intersects the  $1/M_c$  curve. Any intersection represents a shock jump, however the solution cannot jump from a lower to a higher curve.

Figure 2 illustrates our solution procedure, giving the radial dependence of the Mach number  $M$  ((a) and (c)) and  $D$  ((b) and (d)). In Figs. 2(b) and 2(d), the light horizontal lines represent  $1/e$ . The solid and dashed lines correspond to the continuous and broken solutions, respectively. In addition to  $M$ , Figs. 2(a) and 2(c) also give  $1/M_c$ . Figures 2(a) and 2(b) are for  $T = 1.2$  MK, while Figs. 2(c) and 2(d) are for  $T = 1.3$  MK. In both the cases the tube parameters are  $f_\infty = 6$ ,  $f_M = 14$ ,  $r_C = 3R_\odot$  and  $\delta = 0.7R_\odot$ . In view of Figs. 2(a) and 2(b), it is seen that both the curves in Fig. 2(b) exhibit three local extrema, whose locations correspond to the CPs. This follows from that  $dD/dy = 0$  at any CP (see Eq. (4)). Furthermore, the global maximum of  $D$  is attained at the outermost CP, located at  $4.89R_\odot$ . Therefore when the innermost CP is chosen as the SP,  $D > 1/e$  around the outermost CP for  $4.2 \leq r \leq 6.51R_\odot$ . Recalling that  $W(-D)$  is real-valued only when  $-D \geq -1/e$ , one readily understands that in this interval choosing the innermost CP as the SP does not result in a solution to Eq. (1). Figure 2(a) also shows that the curve  $1/M_c$  does not intersect  $M_b$ , indicating the solution to Eq. (1) is unique and is the continuous one.

The situation changes when  $T = 1.3$  MK. Now the global maximum of  $D$  is attained at the innermost CP, located at  $1.75R_\odot$  (Fig. 2(d)). Choosing the outermost CP as the SP leads to that  $D > 1/e$  in the interval  $[1.53, 1.98]R_\odot$ , where there is no solution (Fig. 2(c)). However, two standing shocks are now allowed, since two crossings exist between the curves  $1/M_c$  and  $M_b$ , located at  $2.11$  and  $3.96R_\odot$ , respectively. Hence in addition to the continuous one ( $M_c$  adopting the innermost CP as the SP), two additional solutions exist to Eq. (1): both start with  $M_c$  but one connects to  $M_b$  at the inner crossing, the other connects to  $M_b$  at the outer one.

Although Eq. (1) permits solutions with shocks and time-dependent simulations suggest that these steady-state solutions can be attained, [2,4,5,7] one may still question whether the shock solutions can stand the sensitivity test similar to Ref. [6] which has shown that standing shocks in the solar wind from the center of coronal holes are very unlikely, for the parameter range allowing shock solutions is extremely limited. To see whether the same happens with the streamer geometry, we note that given  $f_\infty$ ,  $f_M$  and  $\delta$ , shock solutions are allowed only in the area bounded by two

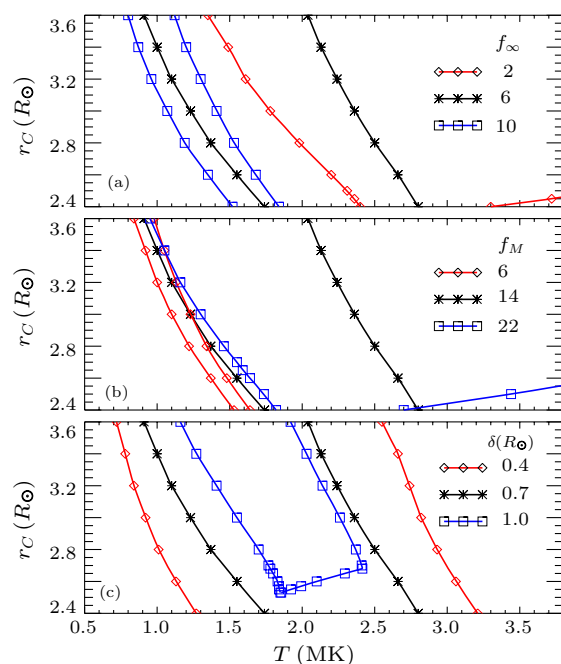
curves in the  $[T, r_C]$  space. Figure 3 presents a series of such curves obtained by varying (a)  $f_\infty$ , (b)  $f_M$ , and (c)  $\delta$  about the reference values  $f_\infty = 6$ ,  $f_M = 14$  and  $\delta = 0.7R_\odot$  (in what follows, the temperatures are in units of MK, i.e.  $10^6$  K,  $r_C$  and  $\delta$  in units of  $R_\odot$ .) Let us first examine the cases with reference values (the black curves connecting asterisks). Figure 3 shows that the area bounded by the two curves is rather broad and with increasing  $r_C$ , both curves are shifted towards lower temperatures, indicating that streamers whose cusps are located higher in the corona are more likely associated with standing shocks. For instance, when the cusp is located at  $3.6R_\odot$ , the slow wind may possess standing shocks as long as  $0.91 \leq T \leq 2.04$ , which actually tends to be lower than the often-quoted values of coronal temperatures. On the other hand, even for the lowest cusp height examined ( $r_C = 2.4R_\odot$ ), the temperature range is  $[1.74, 2.80]$ , still largely compatible with the observational range.



**Fig. 2.** Wind solutions with the streamer geometry for  $f_\infty = 6$ ,  $f_M = 14$ ,  $r_C = 3R_\odot$  and  $\delta = 0.7R_\odot$ . In the left and right columns,  $T$  is 1.2 and 1.3 MK, respectively. Panels (a) and (c) present the  $r$ -dependences of the Mach number  $M$ , while (b) and (d) give those of  $D$ . The solid (dashed) lines represent the continuous (broken) solutions. In (b) and (d), the horizontal lines give  $1/e$  for comparison. Standing shocks are allowed only when the two curves  $1/M_c$  and  $M_b$  intersect.

Figure 3(a) examines the effects of varying  $f_\infty$ . It is seen that with  $f_\infty$  decreasing from its reference value to 2 (the red curves), the  $T$  range allowing shocks increases significantly. Actually for  $r_C \geq 2.5$ , in the examined temperature range there virtually exists no upper bound for shocks to occur. Take  $r_C = 3.6$  for instance. Shock solutions take place as long as  $T \geq 1.35$ . On the contrary, increasing  $f_\infty$  to 10 (the blue curves) makes shocks appear in a much narrower  $T$  range (the width is  $\sim 0.3$ ). The effects of varying  $f_M$  are shown in Fig. 3(b), which shows that increasing  $f_M$  considerably broadens the area allowing standing shocks. For example, with  $f_M$  increasing from 6 to 14, the width along the  $T$ -axis of the area increases from  $\sim 0.1$  to 1.1. When  $f_M$  further increases to 22, this width increases dramatically from  $\sim 0.88$  at

$r_C = 2.4$  to  $\gtrsim 2.9$  for  $r_C \geq 2.6$ . Figure 3(c) shows what happens when  $\delta$  changes. It is seen that increasing  $\delta$  reduces the range of  $T$  where shocks are allowed. For instance, with  $r_C = 3.0$ , this range for  $\delta = 0.4$  ( $\delta = 1$ ) is  $[1.27, 3.21]$  ( $[1.55, 2.26]$ ), while the range for the reference value  $\delta = 0.7$  lies in between. It is interesting to note that for  $\delta = 1$ , at  $r_C \sim 2.68$  the upper bound for  $T$  (the right blue curve) changes its slope dramatically and for  $r_C \leq 2.53$  no shock solutions exist. For  $2.53 \leq r_C \leq 2.68$ , it turns out that on the right of the right blue curve actually no solution exists, since now only two critical points exist and neither of them corresponds to a  $D \leq 1/e$  throughout the computational domain (see Eq. (3)). This is different from the portion  $r_C \geq 2.68$ , where on the right of the right blue curve there does exist a solution which is the continuous one. Putting the three panels together, one may see that for most combinations of tube parameters, the area in the  $T$ - $r_C$  space supporting standing shocks is substantial. Hence with the streamer geometry, standing shocks in the inner slow wind seem physically accessible.



**Fig. 3.** Regions in the  $T$ - $r_C$  space where standing shocks are allowed, shown as the area bounded by the two curves in same color. The results are obtained by varying  $f_\infty$ ,  $f_M$  and  $\delta$  respectively, the rest of the parameters are fixed at their reference values  $f_\infty = 6$ ,  $f_M = 14$ ,  $\delta = 0.7R_\odot$ .

It is not easy to exhaust the possible tube parameters and the consequent changes in shock properties. Instead, let us discuss only the shocks found, examining their detectability. First,  $\delta\rho$ , the density jump relative to the upstream value, is up to 8, a result of the isothermal assumption exceeding the nominal upper limit of 4 for adiabatic gases. As shown in Ref. [13], a  $\delta\rho$  of  $\sim 2.3$  at a standing shock produces an enhancement in the polarized brightness intensity

that is only marginally detectable. A  $\delta\rho$  of 8 certainly makes such detections easier, but one can not say this surely without constructing detailed observables. Second, by conserving angular momentum a coronal shock also produces a discontinuity in the azimuthal flow speed  $v_\phi$ , leading in principle to measurable Doppler shifts in H I Ly  $\alpha$ . However, the jump in  $v_\phi$  turns out  $\lesssim 4$  km/s, discerning which is way beyond the sensitivity of SOHO/UVCS, whose spectral resolution of  $0.23 \text{ \AA}$  translates into about 57 km/s.

The isothermal assumption needs some justification. First, it is not far from reality. The UVCS measurements of the H I Ly  $\alpha$  emission from an equatorial streamer<sup>[14]</sup> showed that the proton kinetic temperature  $T_p$  in the stalk decreases only mildly from 1.45 MK at  $3.6R_\odot$  to 1.3 MK at  $5.1R_\odot$  (their Fig. 3(b)). If the stalk and one of streamer legs are on the same flow tube, then Fig. 4(b) in Ref. [14] shows that  $1.41 \leq T_p \leq 2.09$  MK at  $2.33R_\odot$  (the leftmost two open circles and rightmost two solid ones in their Fig. 4(d)). As for  $T_e$ , the electron-scattered H I Ly  $\alpha$  measured by UVCS yielded a  $T_e$  of  $1.1 \pm 0.3$  MK at  $2.7R_\odot$ .<sup>[15]</sup> Although for a streamer, this value may serve to estimate  $T_e$  in flowing regions at similar heights. Direct  $T_e$  measurements above the distance are sparse. Nonetheless, multi-fluid MHD models indicate that  $T_e$  ranges from 0.8 MK at  $3R_\odot$  to 0.65 MK at  $5R_\odot$  (Fig. 3(d) in Ref. [17]). The mean of  $T_e$  and  $T_p$ , i.e. the temperature  $T$  in this study, is thus about 1.1–1.8 MK at  $2.3R_\odot$  and decreases to  $\sim 1$  MK at  $5R_\odot$ . Furthermore,  $T$  at the slow wind source region is about 0.8–1.2 MK, be the source in a coro-

nal hole or in its neighboring quiet Sun.<sup>[16]</sup> Second, introducing a more complete energy equation, as carried out in Ref. [5] for a coronal-hole flow, will likely *strengthen* rather than weaken our conclusion. The study shows that introducing thermal conduction and two-fluid effects allows for a much broader parameter range supporting standing shocks, compared with isothermal and polytropic computations.

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