Real-time information feedback based on a sharp decay weighted function

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A B S T R A C T

Information feedback strategy, serving as the critical part of intelligent traffic systems, has been treated with growing emphasis. In recent years, a variety of feedback strategies have been proposed. Despite the fact that these strategies have been proved to enhance the traffic efficiency, we find that the road capacity has not been saturated and there is still plenty of room for improvement. Based on the analytic approximations, we found the reason why corresponding angle feedback strategy is superior to weighted congestion coefficient feedback strategy. Given that the sharp decay of the weighted coefficient is the key point, we proposed an efficient feedback strategy called the exponential function feedback strategy (EFFS). We applied it both to the symmetrical two-route model with two exits and that with a single exit. The simulation results indicate that, compared with other strategies, EFFS has decided numerical advantages in average flow, a physical quantity used for depicting the road capacity. Even more importantly, EFFS stands out for its convenient application as well as its fitness for modeling the rugged roads.

1. Introduction

Since the economic bloom and massive urbanization in China, traffic congestion has been getting increasingly worse. The congestion is not only on the traffic network, but also on the Internet. Aiming at solving these problems, many experts from different fields have joined together and contributed their specialties [1–5]. Recently, experts in the field of complex networks have delivered many papers concerning the routing strategy on the Internet [6–12]. In 2006, Wang et al. proposed the static routing strategy based on local information, applicable to the transmission of information packets [6]. Soon afterwards he made some improvements by suggesting the hybrid dynamic routing strategy based on local information [7]. In the same year, Yan et al. proposed the effective routing strategy based on global information, which was of pioneering significance [9]. Based on their work, Ling et al. made further improvements by proposing the dynamic routing strategy based on global information [11]. This strategy is also theoretically proved as the optimum one in the transmission on the Internet. However, with the nodes on the Internet constantly increasing, it is almost impossible to calculate real-time information that is globally based. Therefore, it is mostly the locally based routing strategy that is currently in use. Unlike the Internet, urban traffic networks possess long-term stability. Considering current science and technology, it is utterly applicable to gather real-time traffic information that is globally based, and even easier to gather information on the highway. In recent years, many information feedback strategies aiming at simple urban traffic networks have been proposed [13–27]. The new strategies have made respective contributions to the optimization of road capacity, the congestion coefficient feedback strategy (CCFS) proposed by Wang [15], the weighted congestion coefficient feedback strategy (WCCFS) and the corresponding angle feedback strategy (CABS) proposed by Dong [16,17], to name a few. Nonetheless, when it comes to urban traffic networks, we found that the local feedback based strategy is more effective than the global.

After we compared and analyzed the relative merits of CCFS, WCCFS and CABS, a new strategy was put forward in this paper, which is called the exponential function feedback strategy (EFFS). Combining these three strategies, we obtained the simulation results by applying these four different strategies into a two-route scenario with two exits and one exit, respectively, where vehicles move according to a VDR mechanism [28], instead of an NS mechanism [29] as in previous work [13–21]. From the results and further analysis, we are able to elect the optimal feedback strategy.
2. Related definitions

2.1. VDR mechanism

The VDR mechanism is a new cellular automaton model in analyzing the traffic flow, which considers the velocity dependence of random brake $p$. It was first investigated by Barlovic et al. in 1998 [28]. In this model, the road is subdivided into cells with a length of $\Delta x = 7.5$ m and the cars update their positions and velocities according to the following rules:

1. Acceleration: $v_i(t) \rightarrow v_i(t + 1/3) = \min\{v_i(t) + 1, v_{\text{max}}\}$;
2. Deceleration: $v_i(t + 1/3) \rightarrow v_i(t + 2/3) = \min\{v_i(t + 1/3), d_i(t)\}$;
3. Randomization with probability $p_d(t + 1)$:

\[
 p_d(t + 1) = \begin{cases} 
 p_0, & v_i(t) = 0; \\
 p_d, & v_i(t) > 0. 
\end{cases}
\]

\[
 v_i(t + 2/3) \rightarrow v_i(t + 1) = \max\{0, v_i(t + 2/3) - 1\};
\]
4. Vehicle motion: $x_i(t + 1) = x_i(t) + v_i(t + 1)$.

Here $v_{\text{max}}$ is the maximal velocity of a car, $d_i(t)$ is defined to be the number of empty sites in front of the $i$th vehicle at time $t$, and $v_i(t)$ is the speed of the $i$th vehicle at time $t$. $x_i$ is the location of the $i$th vehicle.

2.2. Two-route scenario

Wahle et al. [13] first investigated the two-route model in which two routes $A$ and $B$ are supposed to be of the same length. In this model there are two types of vehicles: dynamic and static vehicles. Suppose a driver is a dynamic one, he (or she) will make a choice in light of the information feedback. On the other hand, a static one selects routes to enter at random ignoring any advice. The ratio of dynamic and static travelers are $S_{\text{dyn}}$ and $1 - S_{\text{dyn}}$, respectively.

Based on the model, we further regulate that at every time step, a new vehicle arrives at the entrance with the fixed probability $(V_p)$ and chooses either one of the routes in view of the information provided on the broad at the entrance. For example, $V_p = 0.7$ means the chance that vehicles arrive at the entrance at each time step is 0.7. Once a vehicle enters one of the two routes, the motion of it will follow the dynamics of the VDR mechanism and it will be removed when it reaches the exits of the routes.

2.3. Definition of traffic flux

The flux of the routes can be defined as time flux and space flux. In this paper, we use the definition of space flux:

\[
 F = V_{\text{mean}} N / L,
\]

where $V_{\text{mean}}$ represents the mean velocity of all the vehicles on one of the roads, $N$ denotes the vehicle number on each road, and $L$ is the length of two routes.

If $S_{\text{dyn}}$ has been set, the so-called average flux can be calculated as:

\[
 F_{\text{avg}} = \frac{\sum_i f_i}{T \times N}.
\]

Here $f_i$ stands for the flux on the $i$th route at the time $t$ and $T$ stands for the whole time, $n$ is the number of routes.

2.4. Information feedback strategies

Congestion coefficient feedback strategy (CCFS): Each time step, the traffic control center calculates the congestion coefficient of both routes and displays the result on the board according to the locations of every vehicle. The locations are transmitted, received and handled by the navigation system (GPS). When a dynamic car arrives at the entrance, it will accept the advice from the information board and choose the route with the smaller coefficient to enter. The congestion coefficient is defined as:

\[
 C = \sum_{i=1}^{m} n_i^w.
\]

Here, $n_i$ stands for vehicle number of the $i$th congestion cluster. Every cluster is evaluated with a weight $w$, here $w = 2$ [15].

Weight congestion coefficient feedback strategy (WCCFS): Compared with CCFS, WCCFS only differs in the formula for computing the congestion coefficient. Beyond that, procedures of the two strategies are quite identical.

\[
 C_w = \sum_{i=1}^{m} F(y_i)n_i^w = \sum_{i=1}^{m} \left( k \times \frac{y_i}{2000} + 2.0 \right) \times n_i^w.
\]

Here $n_i$ is the same as that explained above. $y_i$ stands for the middle position of the $i$th congestion cluster, and $F(x)$ stands for the weighted function of each route. $w = 2$ as CCFS [16]. $k$ is an undetermined parameter, which affects the traffic flux. Therefore, we have to decide this parameter before applying the strategy to the two-route traffic system. Fig. 1 shows the average flux of applying WCCFS in the two-route scenario with different weight factor $k$, among which $k = -2.2$ leads to the largest average flux.

Corresponding angle feedback strategy (CABS): In this strategy, a pole with a height of $H$ is set at the entrance. Tied to the pole there is a detector. According to the formulas given below, we calculate the angular congestion coefficient of each route, (as shown in Fig. 2):

\[
 C_\theta = \sum_{i=1}^{m} \theta_i^2 = \sum_{i=1}^{m} \left( \arctan \left( \frac{n_{i\text{first}}^w}{H} \right) - \arctan \left( \frac{n_{i\text{last}}^w}{H} \right) \right)^2.
\]
where \( n_{\text{first}} \) and \( n_{\text{last}} \) stand for the position of the first and the last vehicle in the \( i \)th congestion cluster, respectively. \( \theta_i \) stands for the weight (corresponding angle) of the \( i \)th congestion cluster. \( H \) denotes the vertical distance from the point \( T \) to the lane, and in this paper, we set \( H = 100 \) [17]. As for other steps of information feedback, they are identical to those of CCFS.

3. Exponential function feedback strategy

Based on the definitions discussed above, we can reasonably conclude that WCCFS is a strategy based on CCFS, which multiplies the congestion coefficient by a weighted function. When \( k \) is set to be \(-2.1\), the weighted function can be defined as:

\[
-2.1 \times \frac{y_i}{2000} + 2.0.
\]

(6)

Here \( y_i \) stands for the position of the median of the congestion cluster. This is a linear function decreasing with \( y_i \) monotonically. That means as to the congestion clusters of the same size, the one further from the entrance weighs less.

For CAFS, it is also an improvement based on CCFS. To count the congestion coefficient, the author adopted formula (5). Since both \( n_{\text{first}} / H \) and \( (n_{\text{last}} - 1) / H \) are positive, formula (5) approximates:

\[
\arctan \left( \frac{H \times n_{\text{first}}}{H^2 + n_{\text{first}}^2 (n_{\text{last}} - 1)} \right) \approx \arctan \left( \frac{H \times n_{\text{first}}}{H^2 + y_i^2} \right),
\]

(7)

where \( y_i \) and \( n_{\text{first}} \) are the same as that explained above. Formula (7) is also monotonically decreasing with \( y_i \). Considering the characteristics of arctangent function, it decreases more drastically than linear one. Therefore the corresponding angle coefficient decreases more rapidly as the congestion clusters leave the entrance.

Dong et al. have proved that CAFS has advantages over WCCFS in the two-route scenario with one exit [17]. We found the difference between them lay in the fact that the congestion coefficient in CAFS decreases more sharply than that in WCCFS.

Concluding from the two strategies discussed above, we need to design an information feedback strategy where the weight of a cluster drops off when the distances between the vehicles and entrance become large. Meanwhile, the degree of the declination of weighted coefficient is sharper than that in both CAFS and WCCFS. Thus, we suppose that an information feedback strategy where the congestion coefficient decreases exponentially from the entrance to the exit of the lane makes it possible to improve the road capacity.

Exponential Function Feedback Strategy (EFFS): Similar to WCCFS, the traffic center collects the information of vehicles on roads every time step, then computes the congestion coefficients on both lanes according to an exponential function, and displays the result on the board. The dynamic drivers at the entrance will accept the advice from the information board and choose the one with a smaller coefficient. The exponential congestion coefficient is defined as:

\[
C_E = \sum_{i=1}^{m} \exp(-y_i) \times n_i^w.
\]

(8)

Here \( n_i \) stands for the vehicle number of the \( i \)th congestion cluster and \( y_i \) stands for the position of the median of the \( i \)th congestion cluster. Every cluster is evaluated with a weight \( w \) and we set \( w = 2 \).

In the following sections, we will apply the new strategy and the former three strategies into a two-route scenario with two exits to simulate the road conditions and make comparisons. Based on the results, we will select the optimal one.

4. Simulation results

In the simulations, we set the routes of the same length \( L_A = L_B = 2000 \), and the maximum speed of vehicle to be \( V_{\text{max}} = 3 \). The ratio of dynamic drivers is \( S_{\text{dyn}} = 0.5 \). In the VDR mechanism, random break probabilities \( p_0 = 0.3 \) and \( p_d = 0.15 \). The simulation results of the vehicle number, velocity and flux shown here are obtained by 100 000 iterations excluding the initial 95 000 time steps. When it comes to the dependence of average flux on dynamic travelers, we obtain the simulations based on ten times averaged over 100 000 iterations in order to show more precise results.

Fig. 3 reveals the relationship between the average flux \( (F_{\text{avg}}) \) of two routes and the vehicle arrival rate \( (V_p) \) when adopting the four different strategies. In the previous work, the authors assumed that a new car was generated at the entrance of the route every time step, i.e. the vehicle arrival rate is \( V_p = 1 \). From the simulations, we know that this probability has a significant effect on the traffic condition. From the figure we can see when \( V_p < 0.7 \), the average flux is irrelevant to the strategies since the curves of the four strategies almost coincide. In this case, \( 2F_{\text{avg}} = V_p \), meaning all vehicles have free access to the road whenever they arrive at the entrance. However, when \( V_p \) increases above 0.7, considerable changes occur to the curves and the variance of strategies begin to influence the average flux. The growth rate of the average flux (i.e. the slope of the curve) descends because the road is in saturated condition, rendering the vehicles unable to enter the road and having to wait at the entrance. This phenomenon can be conceived as an appropriate model of the traffic condition in large cities during rush hour. Considering that the information feedback strategies do not affect the average flux, it is reasonable to set \( V_p = 1 \).

Fig. 4 illustrates the change of vehicle number of each route versus time when using different strategies. It can be summed up that CAFS is inferior to the other three strategies in terms of the stability. This may be explained by the noise or the initial condition of the simulation. As for the value, the vehicle numbers of EFFS and CAFS are much higher than the others. Some may ask whether the higher vehicle number will lead to more congestion. It shows that while adopting EFFS or CAFS, the number of vehicles will grow from 330 to 400. The length of a route is 2000, so every vehicle can occupy a space of about 5 cells and there will be 3 or 4 cells between two adjoining ones. In this condition, it is rare for congestion to happen. All in all, we believe that EFFS is outstanding for the capacity of the roads.

However, vehicle number is not enough to determine the merits of the feedback strategies. The speed of cars is bound to drop as the vehicle number rises. This can be illustrated from Fig. 5 that shows the relationship between speed and time by using four different strategies. Table 1 demonstrates the mean of the data in Figs. 4 and 5. It clearly manifests the variance of numbers when different strategies are adopted. When we adopt CAFS and EFFS, the average
Fig. 4. (Color online) Vehicle number of each route with (a) CCFS, (b) WCCFS, (c) CAFS and (d) EFFS. The parameters are \( L_A = 2000 \), \( L_B = 2000 \), \( p_0 = 0.3 \), \( p_d = 0.15 \), \( S_{dyn} = 0.5 \), \( H = 100 \) in CAFS, \( k = -2.1 \) in WCCFS.

Fig. 5. (Color online) Speed of each route with (a) CCFS, (b) WCCFS, (c) CAFS and (d) EFFS. The parameters are set the same as in Fig. 4.

Table 1
Demonstrates the mean of the data in Figs. 4 and 5.

<table>
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<tr>
<th>Figure number</th>
<th>Feedback strategy</th>
<th>( N_{avg} )</th>
<th>( v_{avg} )</th>
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<td>Figs. 4 and 5</td>
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<td>2.744</td>
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<tr>
<td>Figs. 4 and 5</td>
<td>Two-route with two exits with WCCFS</td>
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<td>Two-route with two exits with CAFS</td>
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<tr>
<td>Figs. 4 and 5</td>
<td>Two-route with two exits with EFFS</td>
<td>404.453</td>
<td>2.410</td>
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</table>
velocity drops from 2.7 to 2.4, while the vehicle number rises from around 330 to 400. Naturally, here comes the question of what causes the decline of the average velocities in EFFS and CAFS. Fig. 6 answers this question. Presented in Fig. 6 are the histograms of the vehicle number distributions at different velocities, when different strategies are adopted. The data in the figure are the averages of the vehicle numbers at every velocity by every time step, from the 95 000th to the 100 000th. We can distinctly see that the increased 70 vehicles by adopting EFFS or CAFS are mainly contributed by those vehicles with 1 or 0 velocity, compared with other strategies. Since the distributions of vehicle number at other velocities are almost identical among the four strategies, it is evident that the average flux on the road will descend as a whole.

Fig. 7 reveals the relationship between flux and time when adopting different strategies. The stability is about the same, but as for the value, EFFS and CAFS are a little higher than the other three by about 5%. Nevertheless, this advantage is not enough to show its superiority. In the former research, the average flux is regarded as an important standard to weight the capacity of roads, so we use the average flux as criterion.

Fig. 8 demonstrates the change of average flux with different ratios of dynamic drivers. From this, we can see that when adopting CCFS and WCCFS, the average flux goes down if the ratio of dynamic drivers ascends. Thus the more drivers selecting routes according to these strategies, the more unfavorable it is for traffic improvement. What is more important is that when $S_{\text{dyn}} \geq 0.2$, the average flux is even lower than that of $S_{\text{dyn}} = 0$. In view of this fact, it is even better to let the vehicles choose the routes randomly. On the contrary, while employing EFFS or CAFS, the average flux obviously increases with the growing of $S_{\text{dyn}}$. Thus the more drivers picking routes according to these two strategies, the greater the effect the intelligent traffic system displays. Nonetheless, when $0 \leq S_{\text{dyn}} \leq 0.4$, the average flux of EFFS surpasses that of CAFS. When $S_{\text{dyn}} > 0.4$, the curves of the two strategies almost coincide. We find that adopting EFFS results in the largest flux for all the $S_{\text{dyn}}$ and the flux approaches to 0.49 when $S_{\text{dyn}}$ is over 0.4. As for the two-route system, saturation of total flux is 1. If the average is 0.49, according to the definition, the average flux of the two-route system is 0.98, which has been very close to saturation.

Looking into the four figures above, we were amazed to find that the two curves of EFFS and CAFS demonstrate basically the same trend. The average flux of EFFS merely exceeds that of CAFS when $0 \leq S_{\text{dyn}} < 0.4$. What accounts for this similarity is the resemblance of the computational formulas of them. Both the values of the arctangent function ($\arctan(x)$) and the exponential function ($\exp(x)$) increase as $x$ increases. Contrarily, the values of the weighted formula of CAFS ($\arctan(1/x)$) and that of EFFS ($\exp(-x)$) decrease, to the same extent. Apart from the average flux of EFFS exceeding that of CAFS when $0 \leq S_{\text{dyn}} < 0.4$, EFFS has the superiority of convenient application to roads of various shapes. Suppose the road is in the S-shape (as illustrated in Fig. 9), we would not be able to calculate the angular congestion coefficient by employing CAFS. Furthermore, EFFS stands out for another merit. According to the exponential function, the congestion coefficient decreases drastically as the coordinate of congestion cluster increases, thus the congestion coefficient at the end of the road is far less than that at the head of the road, calculated from the exponentially weighted formula. Therefore, some of the information of the end can be omitted in the process of feedback, reducing the workload of the traffic control center.

In consequence, we believe that EFFS is the optimal strategy in the two-route scenario with two exits.

5. Application to the route with a single exit

The two-route model with one exit is the most common one in daily life. It is far more complex than that with two exits. In 2011, Chen et al. proposed a set of rules on the leaving of vehicles [25]:

(a) If the head vehicles on both the lanes at the exit intend to leave the two-route system, the one nearer the exit leaves first;
(b) If the two vehicles are at the same distance from the exit, the one at a faster velocity leaves first;
(c) Under the condition of (a) and (b), vehicles leave randomly.
Fig. 7. (Color online) Flux of each route with (a) CCFS, (b) WCCFS, (c) CAFS and (d) EFFS. The parameters are set the same as in Fig. 4.

Fig. 8. Average flux by performing different strategy versus $S_{dyn}$ into a two-route system with two exits.

We investigated this model with the four strategies the same as we did in the model with two exits. It turned out that when we adopt WCCFS, the average flux reaches the climax with the undetermined coefficient $k = -1.7$. Presented in Fig. 10 are the histograms of the vehicle number distributions at different velocities, when different strategies are adopted. From Table 2 we can see that compared with other strategies, the vehicle number increases by more than 300 after EFFS and CAFS are taken, all falling into those at the velocity of 0, which is illustrated in Fig. 6. This means the frequent occurrence of traffic congestions on the road and the average speed declines almost by half as well. The main cause for this phenomenon is that EFFS and CAFS put a premium on the information at the beginning of the road and omit that at the end. Also, the design of a single exit accounts for the increase of vehicle number. Nevertheless, Fig. 11 shows that the average fluxes of EFFS and CAFS are generally higher than that of the other strategies, particularly when $S_{dyn} > 0.4$. Notwithstanding the merits, the curves of the average flux of these two strategies are nearly straight lines, meaning the invariability of it as $S_{dyn}$ increases. That is, the average flux of drivers selecting routes according to the prompt information practically equals that of drivers selecting routes randomly. However, when it comes to real-life situations, the assumption of absolutely random choices of routes may not be appropriate. For instance, considerable drivers may have their own predilections for routes, some drivers may simply act likewise or the other way around. Taking the impact of individual preference into account, the difference of average flux between drivers with information feedback and those who choose routes at random will emerge.

6. Conclusion

In this paper, we put forward a new feedback strategy called the exponential function feedback strategy. Combined with the
three strategies raised by former researchers, we apply these four strategies into a two-route scenario with two exits and into a two-route scenario with one exit for simulation. We obtain the simulation results about average flux versus $V_p$, and vehicle numbers, speed and flux versus time, as well as the results about average flux versus $S_{syn}$ into the two-route scenario with two exits. These results show that compared with all other strategies, EFFS has distinct advantages in vehicle number and average flux, which is critical in deciding the capacity of the roads. With respect to other quantities such as the stability of velocity and flux, EFFS is not inferior. In addition, EFFS is not only applicable to more complex models, but also relatively convenient to employ. In the symmetrical two-route scenario with one exit, we obtained the simulation results about average flux versus $S_{syn}$. Theoretically the effects of CAFS and EFFS are almost the same as drivers selecting routes at random. However, considering the group psychology and individual preference, completely random choice is hardly possible in reality. Therefore at this point, CAFS and EFFS have their merits in this scenario.

With the development of modern technology, it is not a big deal to realize EFFS. The only precondition for the traffic control center is to collect the information of vehicles on roads via GPS, and then compute the congestion coefficient of each route according to the definition of EFFS. All in all, we think this new strategy is applicable.

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